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PROCEEDINGS OF THE THIRTY-EIGHTH MEETING, HELD AT ANN ARBOR, MARCH 26, 27, AND 28, 1903

PAPERS

EDITED BY THE CHAIRMEN OF THE VARIOUS SECTIONS

GENERAL MEETING

THE DOCTRINE OF INTEREST AS IT AFFECTS EDUCATION.

PRESIDENT L. H. JONES, MICHIGAN STATE NORMAL COLLEGE.

Interest, as it affects the process of education, is a definite state of mind. It is produced by definite causes, and has definite signification suggestive of appropriate courses of action on the part of teacher and student. The possibility of this condition called interest lies in the self-active nature of the human being. The solicitations made by the external world through the senses arouse this self-active spirit, and suggest and stimulate its reaction upon the objects of the external world, and through these objects a reaction upon its own self, enabling it thus to master its own nature on the one hand, and its environment on the other. This continued mastery of the universe by successive steps, led by interest of one kind or another, is the real process of education. The perfection of education would involve knowing all about the capacities, the possibilities of the human spirit, or the self; of knowing all about all the objects of the universe outside the self; and of seeing and using all the helpful relationships of these objects to the self in the process of life development. Now, the spirit masters an object of study by finding out its attributes, and by viewing these attributes in respect of the self, it finds them gratifying or satisfying a felt need of the soul, the self-active character of the human spirit centers more closely its power of discovery upon the object whose attribute has thus gratified the soul,

through the belief in some form that further investigation will disclose in such object still other attributes which will also give satisfaction to the spirit. It is only under motive of some such kind as this that the self-active spirit ever gives its attention in study. Had it no knowledge of self, and had it never experienced the helpful effect of the attributes of other objects upon itself, no possible interest could be felt. Indeed, the infant has no interest. Its first actions are all instinctive. This interest represents a state of mind produced by the discovery in some object other than itself of an attribute which answers to a felt need of the spirit, together with the expectancy that the same object will, on further study, continue such gratification, or even increase it. Of course the state of interest may become exceedingly complex, as, indeed, it usually does; but gratification and expectancy are two permanent elements of interest. The purpose of this paper is, to some extent, to analyze these two elements and to discover if possible, their sources and the method of their development.

When an attribute of an object has been learned by the spirit in some self-helpful relation, and the expectancy of further gratification has been developed in connection with the same object, we are said to be interested in that object, whatever it may be. The class of objects in which we may thus become interested are as numerous and as diverse as the objects of the physical world outside us, as the mental and moral worlds within us, and as the great institutional worlds which we have created; but the interest itself, the gratification of the nature, and the expectancy, are always internal; that is, interest is a purely subjective, or personal matter. When I say that I am interested in a thing, I mean precisely that my mind is apprehending some one or more attributes of this object, and that this attribute or these attributes, through being thus apprehended, are giving satisfaction to me, or promise to do so through some use which I clearly perceive I may make of it or them in what seems to me to be my life development; and that there is in me an expectancy that this object will through this attribute or attributes, or still other of its attributes, continue to give pleasure.

To illustrate, let a horse-shoe magnet be placed before a child who has hitherto known nothing of its capacity or attributes. The first sight of the object has no special interest to him. Why? Because these attributes of the iron, which become apparent on first notice, are not, by their nature, adapted to gratify or answer in any satisfactory way to any of the child's sensibilities. The color is in no special sense fitted to gratify his aesthetic sense; neither does its size, shape or weight gratify any sense of wonder; nor any pleasurable possibility of the soul. No one of these things makes any appeal to any aspirations of the soul; no one of them appeals to any sense of order in the universe; nor do they arouse any feeling of surprise nor any impression of novelty. So although these commoner attributes of the iron are known to the mind, and this same mind understands the capacities of itself, none of these attributes is for the moment calculated to start up any feeling of satisfaction or gratification. So there is as yet no interest felt in the magnet. But this piece of iron has not yet manifested all of its attributes, because as yet no condition has been fixed which would allow

or cause it to do so. Now place some iron filings on the table and pass the magnet near or over them. The magnetism then manifests itself as a power, drawing these filings to the magnet and causing them to adhere with considerable tenacity. Here is an attribute of the iron, which, by its own character, is fitted to minister to the wonder-loving sense of the child, to his innate love of order, to his fundamental notion of cause and effect, to his own feeling of personal power, and withal to his sense of pleasure in the unexpected, not having expected this power in so unpromising an object. This gratification of the wonder-loving propensity begins and a state of interest is soon developed. The feeling is wholly internal to the child; his self-gratification joined to expectancy, and extended outward and connected to the wonderful power of the magnet as its cause. The gratification of the capacities which are involved in the state of interest are mainly pleasures. Indeed, the pleasurable element is always present with expectancy in the true conditions of interest so far as this state of mind is available in education. It is in this respect to be carefully distinguished from those states of fear and terror which fix the attention and develop states of expectancy, but expectancy of other than pleasurable gratifications. In the true state of interest the continued action of the mind is solicited by the object whose attributes cause the expectation, and thus attention is caused through interest. Interest and attention thus react upon each other, each added degree of attention disclosing new attributes in the object, thus increasing interest, and each added degree of interest soliciting and holding greater attention.

Side by side with this emotional phase of interest, this gratification of the sensibilities, the enjoying of those attributes which give pleasure, there runs another stream of gratifications which is a little difficult to define and separate from the foregoing. This is the pure pleasure which always attends the proper action of normal and healthy faculties, whether physical or mental. This is the law of nature. An organ is a faculty that is made for a certain use: the eye is for seeing; the ear for hearing; the hand for handling; the muscle for contracting; the intellect for reasoning; sensibility for loving and hating; the will for deciding. Each power or faculty, physical or mental, has its appropriate function and is adapted to the doing of some special kind of action. There is an uplift to the spirit whenever it cognizes these powers accomplishing the things for which they are intended. Health, happiness, prosperity, the development and satisfaction of each power all lie in doing naturally, freely and fully that which it is adapted and intended to perform. The real pleasure to which I refer here is not always felt as connected with the organ exercised, but is the gratification of the spirit itself at successful action producing commensurate results.

Thus there are two factors which really make up the pleasurable element in the state of interest:

1. The answering of a felt need of the spirit when it has discovered some attribute of an object which answers directly or indirectly a felt need of the self.

2. The sense of satisfaction which the soul feels on finding powers or capacities normally exercised accomplishing intended results.

These two gratifications, supplemented by the state of expectancy which these develop—that the object under consideration, when further studied through these same capacities of the soul, will yield up other attributes which in their turn will gratify the soul—make up the complete state of interest. A state of enthusiastic interest has all three elements raised to a high degree. The interest of children rests about equally on all these sources. Every faculty of the child, his very muscles, eyes, ears, fingers, toes, intellect, sensibility, will, all are in such harmonious relation to one another and to his environment that each movement answers to an instinctively felt need of action, and every new element of knowledge serves a use in satisfying his capacity for enjoyment. In all competitive games the exercise is usually in itself agreeable because of its proper use of the organs and agencies; but the highest sense of pleasure, and hence the deepest interest, centers around the gratification of the ambition to excel, to conquer, to outdo others, to be first among one's fellows, this possibility being the native endowment of every healthy child, derived, no doubt, from the time when his remote ancestors struggled with the wild animals of the forest, vanquishing them by cunning when brute force was lacking. The boy's interest in the game lasts so long as the exercise in itself continues to give pleasure, or the object of enjoyment, victory—remains to lure him on. Both of these pleasurable elements in interest continue strong through early childhood and youth. They are characteristic of childhood. There is action for the sake of action. Action as pure enjoyment, is characteristic of infancy and childhood. Action for the sake of a purpose is a totally different thing, and is characteristic of later youth and manhood, in fact, the result of education and a mark of culture. It has no place whatever in the form of interest which I am now considering. In infancy action is even almost automatic. In most cases the action is stimulated by external forces. It occurs in response to outside stimulus without even conscious satisfaction. A need of the system is met, but not at first a felt need. The child as yet, therefore, has no interest. Soon, however, he awakes to the pleasure of action of both body and mind, and soon he becomes aware of the fact that action answers to a felt need of himself, is a method of self-expression, hence interesting, because gratifying. Soon this action of body and mind secures for him attributes of these objects,—food, rich colors, beautiful clothes, and the response to a felt need of the soul in these directions renders satisfaction. As this element increases, the interest in things through gratifying response which their attributes make to the soul, the actual enjoyment of mere action, while perhaps not growing less, is certainly becoming relatively less noticeable. At any rate, the enjoyment of the knowledge itself, as it answers to a need of the soul, is capable of so great development that it soon becomes a predominant element. Thus it is that we become interested in things, in the actions of others, and whatever furnishes attributes which serve our uses or give us pleasure. We may now be said to have interests. These things which furnish us satisfaction, with which we like to concern

ourselves, which attract us and solicit and receive our attention—attract us through the gratification which these things produce, and the expectancy of the continued gratification which they promise. In all this development of immediate interest, there is little conscious direction of effort toward an end or ideal, but rather spontaneous action of the whole soul on the inspiration of present satisfaction. There is no lapse of time between the action and the enjoyment of such action. In fact, the enjoyment is in the action itself. In the other case, there is no lapse of time between the appropriation of the attribute of the object, and the pleasure which such appropriation gives to the soul. The whole self is bound up in acts and enjoyments. The person has found himself, has expressed his immediate pleasure in this view of the attributes which have yielded him his natural satisfaction. When the number of things in which one thus feels an interest has enlarged through a more extended knowledge of things, the number of one's interests has correspondingly increased. It is manifest, therefore, that one's range of interests is somewhat definitely related to the scope of his knowledge. Since it is impossible that he should feel an interest in anything upon which he has never employed his powers, and whose attributes have never gratified any power of his mind, it therefore follows, so far as he extends his acquaintance with things, he enlarges the possibility of action, and he also enlarges the probability that he will discover more attributes which will furnish him enjoyment, that will increase the number of his interests, that he will find himself in harmonious touch with more and more elements of his environment, and thereby be enabled to live more richly, and to appropriate to his own gratification a larger share of the things of the universe.

Disciples of Herbart in this country have recently laid great stress upon this process of enlarging the range of interests as a part of education, to the end that the educated person may know life on many sides, thus becoming less dogmatical and more liberal, in fact, that he may be large minded, seeing things through the magnifying glass of a large and rich experience. The Herbartians have persistently advocated the doctrine of many-sided interest and diversity of interests. Through the exclusive devotion to this doctrine some of them have laid themselves liable to be charged with being narrow and dogmatic themselves. This estimate is largely due to the unwise and immature statements made by the Herbartians themselves, but chiefly, perhaps, to the narrow interpretation which they have made of the subject. In many cases the whole range of thought has been expended upon this narrow scope of immediate or native interest. It would seem sometimes, from this state of the case, as if the whole work of education were to be restricted to finding what will please the child at once, and then assisting him to satisfy his present propensities. A moment's reflection will show how utterly unworthy this view of education is. The very possibility of education itself lies in the power to create new ideals and therefore the capacity to develop new powers of enjoyment. With these new capacities come new possibilities of education in the discovery of attributes, which, while they do not at once gratify, most completely develop the powers of mind, or tend to answer the higher needs

of the soul. They are not at first so keenly felt, but are capable of being sensitized by education. Thus comes an opportunity of establishing new and higher objects of interest. It is true that modern civilization is chiefly the power to appreciate and use what is of no interest to the savage. The end of education is not to leave man a gratified savage but to develop him into a noble human being. Therefore, the lines of his interest must be changed, and the range of his interests enlarged. Thus it is not alone the business of the teacher to find what will interest the child, but rather to try to interest him in that which is worthy of his capacities and his destiny. Doubtless the process must begin with present interests, but it must terminate in the development of a line of interests which are human, which are worthy of an exalted being, born in the image of God, and capable of developing into fitness for association with noble preson-alities. We must, therefore, look to something more than the gratification of early, or present interests. We must create new ones, must develop new capacities to appreciate, new possibilities of becoming interested in what is worthy in every province of human life and human endeavor. This looks to the development of an interest in a higher grade of attributes, or in the attributes of a higher grade of objects, whose attributes, when mastered, will gratify higher capacities of the soul. In other words, it teaches the child to create for himself ideals of a higher mode of life, and to take an interest in the adaptation of means to the realization of those ideals in a nobler sort of life. Interest in this new mode of life may become as strong and as overpowering as native interest, but its method of development and its law of operation are quite different. The kind of interest which I have thus far been describing is native, immediate, spontaneous. It wells up out of the spirit of the child like the waters of a copious fountain. It comes directly from the gratification which a thing gives to him, and centers itself upon the things whose attributes cause it. If a child should live his childhood and youth under the controlling force of interest of this kind, he would preserve his childlikeness and spontaneity, his happiness and his openness to truth. He would give his whole attention wherever his immediate interest might for the moment lead him. Whatever work he did under such influence would have much the character of play. He might, under its influence, become a great student, and even a great scholar, but hardly a great character. Something more than the influence of present interests is needed to develop strength and stability of character. This native interest is supreme in the childhood of the human being, but it will hardly serve as a proper training for youth and manhood. If one were always to remain a child, there could be nothing further needed than the development for the child intensely this native, immediate interest. Were there never any vicissitudes of life requiring the exercise of a higher virtue and a stronger application of will, there would be little need to develop interest of any higher sort. School education could then consist in making everything as attractive as possible, and all things requiring a real effort in study could be left off. This would, of course, result in no character, only animal life, spontaneous, irrational and unreasoning

life. But manhood and womanhood mean much more than this. They mean the development of character and will, making a thoughtful, manly or womanly person out of every child. For this we must look to a different kind of interest.

As has been stated in an earlier part of this paper, the things in which we may become interested are as numerous and diverse as the objects of the outer world, and of the inner world of mind, and of the institutions which our minds have created. But these classes of things differ widely in one respect, when viewed with reference to the development of interest in them, namely, that some of them interest us immediately because we find in them attributes which cause immediate satisfaction, gratifying powers that are already developed in us, and thus answering to a felt need of our souls. In this case we are interested in these objects as ends of study. Our interest in them is held by the satisfactions which they constantly give us. But there may be other objects that are in no wise interesting to us by reason of any present gratification which their attributes have for us. They do not answer any felt need of the spirit in themselves, but we discover on closer study of their relationships to ourselves that they may be made the means of reaching other things whose attributes do answer our needs, or will answer our needs when we have come into contact with them. Now, it may hardly be said that we at once develop an interest in these objects. It at first seems that we endure them because they are the means by which we reach objects that will interest us. It is the province of education to transform this endurance into a certain sort of interest, which will compel and hold the attention until the mind so masters these objects as to reach through them the objects which will interest the soul through their attributes. This often requires that selected attributes from objects already known to be woven together in memory and thrown upon the canvas of the future in the form of an ideal. The ideal condition, when realized, will furnish attributes which gratify, but the objects which must be used as the means of the realization of this ideal may not gratify the soul which used them. The steps by which one learns to hold his interest in the uninteresting because he has discovered a new relationship of this to the use of his soul is the process of transforming the simple, innocent, irrational, physically active, mentally volatile, child into an intelligent, rational, cultivated person; and between these two extremes lies the large range of our daily teaching, with its necessary deficiencies, its temporary defeats, its final victories; much of it unconscious effort and unconscious tuition, but, as a whole, directed by a more or less well understood philosophy of education.

I know that it is a favorite theory of many that all that is needed is a little time for the child to grow into a cultivated person, that the school should rather be made a place of detention during the growing period. Without ignoring pleasure as a factor, it is sufficient to notice that grown up children are not much superior to young children; but what we need rather is a type of man or woman capable of coping with the growing complexity of modern life. It is true that pleasure is salutatory in its

effect, but its best results accrue only to those who, besides obeying the physical conditions and laws of life, live in the environment of institutions, in exalted companionship with noble human natures, under restrictions which are little appreciated and less liked by the uncultivated man. In other words, the course of education includes the necessary training in the knowledge of communities, in the knowledge of the beneficence of human institutions, the necessity of law and order in the community and self-control in one's self, none of which the savage man in any fair sense appreciates, and many of which are mere means to higher ends. It is thus a large part of education to acquaint the pupil with his spiritual environment, the order of civilization in which he is born, to acquaint him with the immense worth of his spiritual influence, and thus to build up within him a standard of life in harmony with his nature and destiny.

To live up to the high standard of life here outlined, in the midst of a civilization still holding many of the crudities and evils of savage life requires that each of us shall daily do many things which in themselves are not only not pleasurable but which are positively distasteful. In and of itself much of our work is pure drudgery, while much of it requires us to bear large responsibilities, to endure petty annoyances, and to do disagreeable things. It is impossible that we should feel real interest in these things by reason of any gratification of any power of ours by any original attribute of theirs. There is therefore no motive to do these things unless one can be found elsewhere but so related to these acts as to constitute for the time being a valid vicarious interest. The end sought must not only justify but also glorify the means. The contemplation of the ideal end must give a pleasure akin to that felt in its realization, in order that this pleasure shall accompany the doing of the drudgery, gliding at last into the glorious realization of the end achieved by the dull drudgery, till the mind scarcely any longer distinguishes the limits of each and the whole is fused into one glorious sense of achievement. This transformation is complete when this attribute—the relation of means has been clearly seen and the means has accomplished the longed-for end; then the means itself—though it be drudgery—is now loved for one of its own attributes—this very capacity to achieve desired ends—then is *immediate* interest in the means itself developed in place of vicarious or mediate interest.

Happy is he in life who can so live that the effulgent glory of his ideal life is thrown backward till it lights up all the pathway of his actual life. His ideal is the magnet pole of his life. He will drudge for ten hours per day if need be in order that he may found his ideal family life and keep it sweet and pure under the shadow of his own vine and fig tree. He will march with steady step to the cannon's mouth, at the call of his patriotic ideal, counting life and limb as mere incidents in the series of movements by which civil and religious liberty are established. He will council together with his neighbors, foregoing his personal preference to the end that the social order may be unbroken. His interests are so set in the best things that he cannot stoop to the mean or low; and the fine sense of gratification coming from the realization within himself of a high

grade of manhood compensates for laborious efforts and frequent disappointments in external purposes. The perfection of culture is to think clearly, to aspire nobly, to drudge cheerfully, to sympathize broadly, to decide righteously and to perform ably. The undeveloped child can do none of these things. The undeveloped germ of the possibility to do these things is his by native endowment. The province of eduation lies between these two extremes. To understand the philosophy of education we must study the child as he is, to get our point of starting. But we must study the possibilities of man, as he has expressed himself in history, literature, art and achievement, to get the other point which sets our trend and gives direction to our educational effort.

FRENCH SECTION

TO WHAT ACCOMPLISHMENT SHOULD THE TWO YEARS' COURSE IN HIGH SCHOOL BE DIRECTED?

HELEN FARRAND NAUMANN, PORT HURON.

In arranging a plan for French work in high schools two groups of pupils must be considered. First: those fitting themselves for entrance to a university, and second: pupils whose systematic training will end with their high school course. The former must be well provided for, or a school can claim no place on the "diploma list," while the best interests of the latter, constituting the majority in each class, should be equally our care. This diversity of purpose on the part of students, however, presents to my mind no added difficulty. For I would instruct both classes of pupils in exactly the same way. Not that the same method could be advantageously employed in the lower grades, or outside of schools in many instances. Immaturity of mind in the one case, and a lack of adequate technical preparation in English in the other would render the adoption of such a course unwise. But to those who have entered the high school after thorough work in the lower grades grammatical terms should be perfectly familiar, logical presentation of grammatical principles intelligible, and, in general, the grammatical path to the fair land of France in speech and literature the most direct one. But I would make this path a broad and beautiful avenue, well provided with all that can render the journey pleasant, relieve fatigue, and prevent monotony, while not detaining the traveller too long nor deflecting his steps.

My conviction that even those high school pupils not contemplating a university course should be trained on strictly scientific lines, is based

upon personal observation and experience. I have seen the efficacy of various methods practically tested on French soil. For greater clearness, I will separate into four classes those American students of French whom I have observd abroad.

I. Those with *no* previous knowledge of French.

II. Victims of the exclusively "conversational" method.

III. Students who had pursued the laborious course formerly traversed in studying ancient languages.

IV. Those who, with careful study of grammatical principles, had acquired an ability to apply such principles through systematic drill in conversation, dictation, composition, etc.

I have noted that beginners soon overtake those instructed in America by the conversational method exclusively, and that students possessing only theoretic knowledge find themselves utterly helpless regarding the spoken language. On the other hand, the results of a course of study furnishing a thorough elementary knowledge of grammar strengthened by constant practical application are in the highest degree satisfactory. A few weeks' stay in Paris then suffices to equip the student with a supply of words and phrases adequate for all needs, whether his aim be science, art, or general culture. Each acquisition finds its allotted place, all resting on a firm and broad foundation.

It is true that only a small proportion of our students will test their knowledge of French in Paris. But it will be conceded that what best prepares one for further study or social intercourse in France constitutes likewise the best preparation for any later use of the language.

Since the claims of grammar for an important place in our course have been now presented, it remains to be considered what should be the scope of work in this line. In my opinion, we must limit ourselves to the essential, the eminently practical, while taking care that our foundation be fitted for permanency. Historical grammar, and syntactic intricacies should be reserved for later study, even in longer courses. In the two years' course, lack of time would prevent their introduction; but aside from this, inaccuracy and timidity of memory often result from too early invasion of these grammatical provinces. This reflection would not preclude, however, the occasional explanation of a derivation (especially useful to Latin pupils as an aid to correct orthographic accent) or the notice of English-French cognates. Such observations tend to increase interest, and prove to pupils the correlation of their studies—an important point.

The choice of a text-book in grammar is a serious thing, even though it be used simply as a manual. With eagerness I examine each new aspirant for honors in this field that reaches me, only to lay it down with that feeling at my heart occasioned by "hope deferred." With due appreciation of admirable points in many new grammars, I have not yet seen what seems to me the ideal book for English-speaking pupils. A thought expressed in a recent article on modern architecture might find an application here, in many cases:—"Architects now originate simply for the sake of novelty, not in order to produce something better, but merely something unfamiliar."

My objection to most of the new grammars is that their plan is not sufficiently scientific. They do not furnish the foundation required by all varieties of students. Other text-books with excellent plan omit any presentation of French equivalents for grammatical terms, thus necessitating the consumption of much extra time in case French is to be largely the language of the class-room.

Of the grammars with which I am acquainted, I prefer Whitney's as a text-book for beginners, owing to its logical presentation of most of the essential principles, and its systematic arrangement. Retaining its frame work, the material parts may be adapted and elaborated to suit one's own ideas. In my opinion, an equivalent to Part I. of this grammar furnishes sufficient knowledge of principles for our course. The eight long years spent on French grammar in the schools of Germany do not produce the practical results that would satisfy me as a teacher, and such drudgery may engender great distaste for the subject. Who knows but that in the irksomeness of this eight years bondage to "dem Kleinen und dem grossen Ploetz" may lie the hidden cause of many a modern German's lack of love for his French brother? But a genuine "Prussian drill" in grammatical forms and fundamental rules cannot be dispensed with in any thorough course.

All knowledge of rules or of word-forms, however, avails little without a reliable vocabulary. The most skillful cook cannot make bread without flour. Therefore, an absolute *possession* of the most commonly used words should be one of our requirements. To stimulate endeavor, and accomplish accuracy in this regard may be recommended an occasional "vocabulary down," conducted on a plan similar to that of the old fashioned "spell-down." In my experience, this exercise rather saves than consumes extra time in the end, and "idiom-downs" form another profitable pastime for pupils somewhat advanced.

The entrance requirements of our state university most wisely emphasize a practical knowledge of French rather than the reading of a long list of texts. I speak feelingly on the subject of text-lists, for the diversity of those required for entrance into various colleges has repeatedly caused me great perplexity. In gathering data for a paper presented by me at a conference held at Chicago University a few years ago, I ascertained that if, in a class of four, each member were preparing for one of four certain colleges respectively, the class would be obliged to read twenty-seven different texts, to say nothing of other class work. A special list of texts for the two years' course seems to me inexpedient, although the reading should be confined to modern authors, and might well include a comedy. Quality and not quantity of work in reading is our most important consideration. Translating into English, moreover, is a matter that almost takes care of itself. It has a place in every good course, and its usefulness is apparent. But, with many instructors, far too much time is devoted to it. Only on rare occasions is it necessary to require the translation, in class, of a review lesson. Its substance may be narated by pupils consecutively in well-rounded French sentences, or may be reproduced in answer to questions from the

instructor, or from pupils in turn. Conversation among students in class by way of query and answer may well begin as soon as one tense of the first verb has been learned, in addition to a few nouns. Pupils are delighted at the achievement of these first tottering steps, and as surprised as was Molière's M. Jourdain when he discovered that he could talk prose. These slight successes also bring confidence, a most important aid in acquiring a foreign tongue, particularly in the case of boys and girls of high school age—that period of blushing and stammering self-consciousness. This same self-consciousness is the most formidable hindrance to correct pronunciation. There are individuals in every class who must be drilled by themselves. They are never willing to risk possible ridicule by attempting to roll an "r" or to produce one of the nasal sounds in the presence of classmates untill such feats have become quite safe and easy. In these private rehearsals I have more than once discovered a "star-pupil," thus reaping a speedy reward for my extra labor. Each correct enunciation of a French sound, each correct utterance of a French expression, is a firm step in advance, for our vocal organs, no less than our brains, are wondrously influenced by habit. The difficulty in securing sufficient practice of this kind in large classes may be met by concerted repetition, sometimes with closed books, and even with closed eyes, in order to insure complete attention to sound.

Thus, step by step, our student acquires the ability to pronounce correctly and fluently even in sight-reading, as well as to recognize readily French words and expressions when spoken. Dictation exercises become a pleasure, and confidence in himself brings increased zest for further study.

In composition work, which should be one of the prominent features of our plan, modern methods have produced a most valuable change, and what was often drudgery in former years has become an agreeable task. No longer does the student dread composition days. In fact, repeatedly individual pupils have requested me to oversee extra work in this line, asserting that "it is so much fun to write the exercises." If, immediately upon the acquisition of barely sufficient material, the student is required to write original sentences during the recitation period—a little privilege that I often attach to the speedy completion of assigned work—or to prepare illustrative expressions as part of his work in learning idioms, a beginning of composition work is made, and the transition to connected sentences follows naturally. In my monthly tests, after the first few weeks, all questions are expressed in French, and not one English word appears in the answers to these questions, or, in fact, on any examination paper from the beginning of work. I require no translation from French into English in any examination.

To one who regrets the present rareness among students of an old-fashioned memory, it is gratifying to note the growing encouragement given to memorizing choice bits of French verse. The importance of this exercise should exact a definite place for it in our plan of work. It has long been my custom to have second year pupils memorize with distributed parts whole acts from some modern comedy read by them. This never fails to interest a class, and the results are always more than satisfactory.

What may we then secure by the two years' course? I answer: A correct and fluent pronunciation, together with a readiness in recognizing French spoken words; a good working vocabulary and acquaintance with common idioms; the ability to translate, into or from French, language of ordinary difficulty, and to write easy French off-hand; and a practical knowledge of grammatical principles. To these items should be added a cursory acquaintance with France, her life and literature.

Moreover, without losing sight of our aim, we must not fail to awaken and maintain on the part of students a keen interest in their work. When interest flags, progress is slow and awkward. To popularize the study of French, to increase the number of pupils pursuing it is by no means the least important part of our task. Fortunately for us, language work may be presented in endless variety if we choose to avail ourselves of its possibilities.

MATHEMATICAL SECTION

REPORT OF THE BRITISH ASSOCIATION COMMITTEE ON THE TEACHING OF ELEMENTARY MATHEMATICS.

PROFESSOR W. W. BEMAN, UNIVERSITY OF MICHIGAN.

The report of the British Association Committee on the Teaching of Mathematics, as presented by its Chairman at the Belfast meeting in 1902, is as follows:

In submitting their present report, the committee desire to point out that this is not the first occasion on which the British Association has attempted to deal with the teaching of elementary mathematics. About thirty years ago, a similar body was appointed to consider a part of the subject, viz., "the possibility of improving the methods of instruction in elementary geometry," and two reports were presented, one at the Bradford meeting in 1873 (see the Report volume for that year, p. 459), the other at the Glasgow meeting in 1876 (see the Report volume for that year, p. 8).

The two reports advert to some of the difficulties that obstruct improvements in the teaching of geometry. One of these is alleged to be "the necessity of one fixed and definite standard for examination purposes"; apparently, it was assumed that this fixed and definite standard should not merely be required from all candidates in any one examination, but also be applied to all examinations throughout the country. In order to secure the uniformity thus postulated, the committee thinking that no text-book had been produced fit to succeed Euclid in the position of authority, and deeming it improbable that such a book could be produced by the joint action of

selected individuals, suggested the publication of an authorized syllabus. In their second report they discussed the merits of a particular syllabus—that of the Association for the Improvement of Geometrical Teaching; but, in spite of such commendation as was then expressed, the syllabus has not been generally adopted.

It is still true that (in the words of the former Committee) "in this country at present teaching is guided largely by the requirements of examinations." For some time to come, the practice of the country is not unlikely to allow examinations to retain at least a partial domination over teaching in schools. Accordingly, it seems to be a preliminary requisite that examinations should be modified; and, where it is possible, these modifications in the examinations should leave greater freedom to the teacher, and give him more assistance than at present.

On the other hand, there is a tendency in this country whereby, in such matters as teaching and examination, the changes are only gradually effected, and progress comes only by slow degrees. Accordingly, the general recommendations submitted in this report are such that they can be introduced easily and without any great alteration of the best present practice. It is the hope of the Committee that the recommendations, if adopted, will constitute merely the first stage in a gradual improvement both of teaching and of examinations. For the most part only broad lines of change are suggested; this has been done in order to leave as much freedom as possible to teachers for the development of their methods in the light of their experience.

IS UNIFORMITY IMPERATIVE?

The Committee do not consider that a single method of teaching mathematics should be imposed uniformly upon all classes of students, for the only variations then possible would be limited by the individuality of the teacher. In their opinion, different methods may be adopted for various classes of students, according to the needs of the students, and corresponding types of examinations should be used.

It is generally, if not universally, conceded that a proper training in mathematics is an important part of a liberal education. The value of the training depends upon the comprehension of the aims of the mathematical subjects chosen, upon the grasp of the fundamental notions involved, and upon the attention paid to the logical sequence of the arguments. On the other hand, it is freely claimed that, in the training of students for technical aims such as the profession of engineering, a knowledge of results and a facility in using them are more important than familiarity with the mathematical processes by which the results are established with rigid precision. This divergence of needs belongs, however, to a later stage in the training of students. In the earliest stages, when the elements of mathematics are being acquired, the processes can be substantially the same for all students; and many of the following recommendations are directed towards the improvement of those processes.

TEACHING OF PRACTICAL GEOMETRY.

The former Committee recommended (and the present Committee desire to emphasize the recommendation) that the teaching of demonstrative geometry should be preceded by the teaching of practical and experimental geometry, together with a considerable amount of accurate drawing and measurement. This practice should be adopted, whether Euclid be retained, or be replaced by some authorized text-book or syllabus, or if no authority for demonstrative geometry be retained.

Simple instruments and experimental methods should be employed exclusively in the earliest stages, until the learner has become familiarized with some of the notions of geometry and some of the properties of geometrical figures, plane and solid. Easy deductive reasoning should be introduced as soon as possible; and thereafter the two processes should be employed side by side, because practical geometry can be made an illuminating and interesting supplement to the reasoned results obtained in demonstrative geometry. It is desirable that the range of the practical course and the experimental methods adopted should be left in large measure to the judgment of the teacher; and two schedules of suggestions, intended for different classes of students, have been submitted to the Committee by Mr. Eggar and Professor Perry respectively, and are added as an Appendix to this Report.

SHOULD THERE BE A SINGLE AUTHORITY IN GEOMETRY?

In the opinion of the Committee, it is not necessary that one (and only one) text-book should be placed in the position of authority in demonstrative geometry, nor is it necessary that there should be only a single syllabus in control of all examinations. Each large examining body might propound its own syllabus, in the construction of which regard would be paid to the average requirements of the examinees.

Thus an examining body might retain Euclid, to the extent of requiring his logical order. But when the retention of that order is enforced, it is undesirable that Euclid's method of treatment should always be adopted; thus the use of hypothetical constructions should be permitted. It is equally undesirable to insist upon Euclid's order in the subject-matter; thus a large part of the contents of Books III. and IV. could be studied before the student comes to the consideration of the greater part of Book II.

In every case, the details of any syllabus should not be made too precise. It is preferable to leave as much freedom as possible, consistently with the range to be covered, for in that way the individuality of the teacher can have its most useful scope. It is the competent teacher, not the examining body, who best can find out what sequence is most suited educationally to the particular class that has to be taught.

A suggestion has been made that some Central Board might be instituted to exercise control over the modifications made from time to time

in every syllabus issued by an examining body. It is not inconceivable that such a Board might prove useful in helping to avoid the logical chaos occasionally characteristic of the subject known as Geometrical Conics. But there is reason to doubt whether the authority of any such Central Board would be generally recognized.

Opinions differ as to whether arithmetical notions should be introduced into demonstrative geometry, and whether algebraic methods should be used as substitutes for some of the cumbrous formal proofs of propositions such as those in Euclid's Second Book; for opinions differ as to the value of strictly demonstrative geometry, both for training and for knowledge. Those teachers who do not regard algebraic methods as proper substitutes for geometric methods might still use them, as well as arithmetical notions, for the purpose of illustrating a proposition or explaining its wider significance. It is the general opinion of the Committee that some association of arithmetic and algebra with geometry is desirable in all cases where this may be found possible; the extent to which it may be practised will depend largely upon the individual temperament of the teacher.

Every method of teaching demonstrative geometry has to face the difficulties inevitably associated with any complete and rigorous theory of proportion. In the opinion of the Committee, not merely is Euclid's theory of proportion unsuited for inclusion in elementary work, but it belongs to the class of what may be called university subjects. The Committee consider that the notion of proportion to be adopted in a school course should be based upon a combination of algebraical processes with methods of practical geometry.

EXAMINATIONS IN GEOMETRY.

As regards examinations in geometry, the Committee consider that substantial changes in much of the present practice are desirable. In most, if not in all, of the branches of mathematics, and especially in geometry, the examination ought to be arranged so that no candidate should be allowed to pass unless he gives evidence of some power to deal with questions not included in the text-book adopted. Such questions might comprise riders of the customary type, arithmetical and algebraical illustrations and verifications, and practical examples in accurate drawing and measurement. The Committee consider the latter of particular importance when the range is of an elementary character; some influence will be exercised upon the teaching, and some recognition will be given to the course of practical geometry that should be pursued in the earlier stages.

ARITHMETIC AND ALGEBRA.

The Committee are of opinion that, in the processes and explanations belonging to the early stages of these subjects, constant appeal should be made to concrete illustrations.

In regard to arithmetic, the Committee desire to point out what has been so often pointed out before, that, if the decimal system of weights and measures were adopted in this country, a vast amount of what is now

the subject-matter of teaching and of examination could be omitted as being then useless for any purpose. The economy in time, and the advantage in point of simplification, would be of the greatest importance. But such a change does not seem likely to be adopted at present, and the Committee confine themselves to making certain suggestions affecting the present practice. They desire, however, to urge that teachers and examiners alike should deal with only those tables of weights and measures which are the simplest and of most practical use.

In formal arithmetic, the elaborate manipulation of vulgar fractions should be avoided, both in teaching and in examinations; too many of the questions that appear in examination papers are tests rather of mechanical facility than of clear thinking or of knowledge. The ideas of ratio and proportion should be developed concurrently with the use of vulgar fractions. Decimals should be introduced at an early stage, soon after the notion of fractions has been grasped. Methods of calculation, accurate only to specified significant figures, and, in particular, the practice of contracted methods, should be encouraged. The use of tables of simple functions should be begun as soon as the student is capable of understanding the general nature of the functions tabulated; for example, the use of logarithms in numerical calculation may be begun as soon as the fundamental law of indices is known.

In regard to the early stages of algebra, the modifications (both in teaching and in the examinations) which are deemed desirable by the Committee are of a general character.

At first, the formulæ should be built on a purely arithmetical foundation, and their significance would often be exhibited by showing how they include whole classes of arithmetical results. Throughout the early stages, formulæ and results should be frequently tested by arithmetical applications. The arithmetical basis of algebra could be illustrated for beginners by the frequent use of graphs, and the practice of graphical processes in such cases can give a significance to algebraical formulæ that would not otherwise be obtained easily in early stages of the subject.

In passing to new ideas, only the simplest instances should be used at first, frequent reference still being made to arithemetical illustrations. Advance should be made by means of essential development, avoiding the useless complications of merely formal difficulties which serve no other purpose than that of puzzling candidates in examinations. Many of the artificial combinations of difficulties could be omitted entirely; the discussion of such as may be necessary should be postponed from the earlier stages. Teachers and examiners alike should avoid matters such as curious combinations of brackets; extravagantly complicated algebraic expressions, particularly fractions; resolutions of elaborate expressions into factors; artificially difficult combinations of indices; ingeniously manipulated equations, and the like. They have no intrinsic value or importance; it is only the mutual rivalry between some writers of text-books and some examiners that is responsible for the consideration which has been conceded to such topics.

GENERAL REMARKS.

If general simplification either on these or on similar lines be adopted, particularly if graphical methods are freely used, it will be found possible to introduce, quite naturally and much earlier than is now the case, some of the leading ideas in a few subjects that usually are regarded as more advanced. Thus the foundations of trigonometry can be laid in connection with the practical geometry of the subject-matter of the Sixth Book of Euclid. The general idea of co-ordinate geometry can be made familiar by the use of graphs, and many of the notions underlying the methods of the infinitesimal calculus can similarly be given to comparatively youthful students long before the formal study of the calculus is begun.

ON THE RELATION BETWEEN MATHEMATICAL RESEARCH AND SECONDARY INSTRUCTION.*

PROFESSOR E. R. HEDRICK, UNIVERSITY OF MISSOURI.

I consider it a great privilege to address the Michigan Schoolmasters' Club, and particularly upon a subject of such import. The breadth of the topic is so great, however, that I feel myself quite unable to do more than point out a few of the many questions which naturally arise, and suggest to you the advisability of their serious consideration. That I should attempt to pass final judgment upon matters of such difficulty, would indeed be presumptuous; my only regret is that I cannot be present in person to learn the views of you who come into such direct contact with the practical side of the questions of which I am to speak.

That the enormous strides made in the study of pure and applied mathematics in the last score of years, must eventually react upon the instruction in the elementary mathematics of the secondary schools and of the first two years of college, seems certain. Just what effect this reaction will have upon such instruction is now very problematical; and without careful consideration and coöperation on the part of earnest teachers throughout the country, the good effects, to which we may reasonably look forward, may be long delayed, or rendered temporarily abortive. On the other hand, it is of vital importance that intelligent teachers should guard against the tendency to overdo any attempted reform, and see that its force is not lost in "fad-isms".

It is for these two reasons that I wish to suggest and explain the following tentative propositions relative to the teaching of mathematics in high schools and in the first two years of college. I sincerely trust that their promulgation will fix attention upon the subject, and that it will provoke a most lively criticism. I would then suggest:—

*Read by Professor Beman, in the absence of the writer.

(1°) That an effort be made to appreciate and to apply in elementary teaching, the results of modern research in pure and applied mathematics;

(2°) That to this end, to avoid overdoing the matter, the teacher himself should attempt to master those features of the subject which apply to elementary teaching; but that the effect upon the student should be left to the unconscious influence of the teacher's improved points of view, rather than that any radical departures be made immediately, from the material actually taught at present, or from the present methods of instruction;

(3°) That more attention should be paid, however, to the physical and other applications of the subjects taught; and that the various branches should be brought into much closer correlation, or, in certain cases, actually intermingled with one another;

(4°) That a system of laboratory instruction, on a very modest scale, should be introduced gradually; and that boards of education and others instrumental in the arrangement of curricula, be impressed that the advantage to be derived from mathematical exerices over pure class-room drill, is equal to the enormous advantage recently realized in the instruction in physics and chemistry, by the general introduction of laboratory instruction in those subjects;

(5°) That the requirements for admission to college be adapted so as to foster, or at least so as to permit of, such changes in instruction; preferably by the adoption of alternate requirements, which could be satisfied by either method of instruction.

In the discussion and explanation of these theses, it will be enlightening to consider for a moment the history of mathematics in America.

Prior to 1875, aside from one or two illustrous examples, including at least Benjamin Peirce, the advances made on the continent of Europe in the study of pure mathematics, were practically unkonwn, or neglected, in this country. Up to about this time, original work in mathematics by Americans was almost unheard-of, and the knowledge of that done by foreigners was far from general, even among men in the highest position in the American mathematical world.

After about 1875, due to the increasing prosperity and settled condition of the country, more attention began to be paid to the purely scientific side of mathematics, and such a movement, once inaugurated, necessarily bore the fruits of its own logical justification. Today, contrasted with the condition of mathematical knowledge in America in 1875, the situation can fittingly be described as phenomenal and phenomenally encouraging. In almost every large university in the country, and in surprisingly numerous smaller colleges, men are to be found, in ever increasing numbers, whose whole lives are consecrated to the development of pure and applied mathematics; and whose original contributions to the theory commence to be of no mean order, even when compared with the productions of the schools of continental Europe, backed by their hundred years of standing—for modern mathematics is now practically in the beginning of its second century. That these men appear otherwise normal, sane, and appreciative of other sciences, and that they are surely not working for pecuniary gain, leads us to feel

that behind the veil of apparent uselessness and impracticability, must lie an enchanting field of actual reality and fruitfulness, which charms them to their self-imposed tasks, and inspires them in their work.

The American Mathematical Society, non-existant in 1887, now has over four hundred members, bound in a union of fraternity and of enthusiastic sympathy, which no scientific society, at home or abroad, can surpass. This society now publishes two journals, the *Bulletin,* and the *Transactions,* which are recognized, even in Europe, to be among the leading mathematical journals of the world. And at least two other mathematical journals of excellent reputation are published in this country: the *American Journal of Mathematics,* published by Johns Hopkins University; and the *Annals of Mathematics,* published by Harvard University. What is of more consequence, the papers published by these journals are, almost exclusively, the productions of American mathematicians—teachers in our schools and colleges.

Such is, in brief, the history of mathematics in America during the last thirty years, constantly increasing in volume and importance; the last fifteen years cover the history of the American Society; while only in the last five years have the courses in instruction in higher mathematics in our Graduate Schools approached anything like equality with the work in Germany, France, and Italy. It is therefore not surprising that modern mathematical research has not as yet generally affected the elementary instruction in mathematics in American secondary schools; but the time is surely ripe for the serious discussion of this inevitable result.

I propose to discuss briefly a few of the points at which recent investigation touches upon elementary instruction; and, in doing so, I shall make use of ideas which are already prevalent abroad, and of various papers which have recently appeared in American and in European journals, treating the subject of elementary mathematical instruction from a similar standpoint.

Among the questions which arise, the discussion of the axioms underlying geometry and algebra has recently attracted the most widespread attention. I do not agree with those enthusiasts who would force upon the unprepared mind of the young student, ideas of such comparative complexity as those at the basis of these discussions. But it does seem fitting that the *teachers* of mathematics should know and appreciate the excellent work recently done on these subjects, to the end that the teachers themselves be not ignorant of the present state of knowledge, and that they may be able to avoid the formation of wrong conceptions in the minds of the pupils. This can all be done, and without changing the *form* of instruction at all; for, while the matter is not suitable for direct presentation to the student, it is perfectly possible for an intelligent teacher, even though his mathematical education has been restricted to the most elementary college courses, to so familiarize himself with the essential ideas as to avoid forms of expression which would induce wrong conceptions in the student's mind. The study of the axioms of mathematics, in its modern conception, probably started with Gauss, in Germany, about 1800. It was he, and his followers, who showed conclusively the impossibility of proving the axiom of parallels from the

other axioms usually assumed. Even this work was not completed, however, until the middle of the century. The existence and the logical character of geometries in which the Euclidean axiom does *not* hold, and the application of such studies to highly practical problems, are still fruitful subjects of investigation.

But it is even more recently that the greatest light has been thrown on the subject of the axioms of mathematics, and their real significance. Beginning with the work of Pasch, in Germany, and Veronese, in Italy, this newer view found its most brilliant exposition in the now famous treatise on the foundations of geometry*, published in 1900, by Hilbert, in Germany. This book is now fortunately available in English, in a fair translation**; and it should be in the hands of every sincere teacher of geometry—not for his pupils, for they cannot understand its true meaning, but for the teacher's own guidance and information. In obtaining the English translation, the list of corrections, now in preparation, should be secured.

The purposes of such work cannot be explained hastily; it is necessary for one to study in private these apparently trivially simple developments, and to try to discover the real reason for the enormous importance which is now universally attached to them. Let me mention a few points which are now accepted as fundamental canons.

(1°) *The logical consequences of a set of axioms are theorems, which must hold for any set of "things" for which the axioms themselves hold.*

(2°) *In a logical proof, no appeal whatever can be made to the intuition, since the theorems do not depend on the nature of the "objects" employed, except in that the axioms are satisfied.* But *in elementary instruction* intuition should be employed *constantly*: the only thing desired is that the *teacher himself* should not lose sight of the nature of a *proof, in a strictly logical sense.* That assumptions made in elementary instruction are quite allowable, will be seen from No. (5).

(3°) *The only absolutely necessary characteristic of a set of axioms is that the statements made shall in no wise* CONTRADICT *one another.* To show that a given set of axioms does not contradict itself, a *proof* is necessary, since we have no means of judging directly; and this proof is usually very difficult. *Still no proof should be given to the student.* But the *teacher himself* should try to grasp the proof given by Hilbert, that *the ordinary axioms of ordinary geometry are,* in this sense, *compatible.*

(4°) *It is further desirable (but in no sense a logical necessity) that the set of axioms assumed should* (a) be as *simple* as possible, (b) contain as few *redundancies* as possible, (c) correspond with reasonable accuracy to our *ordinary* conceptions and ordinary intuitive ideas, derived from experience.

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*D. Hilbert, *Grundlagen der Geometrie,* Leipzig, Teubner, 1899, 8 vo., 92 pp.

**D. Hilbert, *The Foundations, of Geometry,* Translated by E. J. Townsend, The Open Court Publishing Co., Chicago, 1902, 8 vo., 132 pp.

(5°) *It is permissible, in elementary instruction, to introduce* AS AN (AXIOM *any statement whatever, which can be proved from the other axioms assumed, i. e., any theorem; or indeed any other statement which does not contradict the original set of axioms, whether or not it is a consequence of them.* This seemingly revolutionary statement is surely justified in the light of these researches, and it should be made clear to every teacher of mathematics. The teacher should of course use proper judgment in assuming theorems in this way without proof, and he should convince *himself,* of course, that the assumptions *are* theorems, or at least, that they are compatible with the axioms. Otherwise their assumption is justifiable on exactly the same grounds as is the omission of the general proof of compatibility.

(6°) *An axiom is by no means a "self-evident truth," nor a "proposition which cannot be demonstrated".* The first of these usual statements corresponds to the notion that intuition is infallible; the second asserts that the system used is entirely free from redundancies. The teacher should realize that *an axiom is merely an arbitrary assumption, which can be altered at the will of the operator;* and that *a set of axioms is any set whatever of such arbitrary assumptions, whose sole necessary characteristic is their compatibility.* We choose from all possible sets, for purposes of elementary instruction, one which is *simple,* and which *corresponds to a reasonable extent with our rough conceptions* gained from necessarily crude experience, for the science under discussion. Care is necessary, logically, only in the proof that the axioms are compatible.

But it is not intended that this short paper should form any complete exposition. The reasoning which leads to the above facts must be carefully gone through before one can render an intelligent judgment. I shall be satisfied if I have directed attention to the subject; for, while the bald statements are at times startling, they are the inevitable result of any thoughtful and intelligent consideration.

I would mention, as enlightening upon the whole question, besides the work already referred to, the following books and papers, which should be in every college library, and accessible to teachers generally:—

Karl Pearson, *The Grammar of Science,* second edition, London, Black, 8 vo., 548 pp.

B. A. W. Russell, *The Foundations of Geometry,* Cambridge University Press, 1900, 8 vo., 201 pp.

Ernst Mach, *The Science of Mechanics,* translated by T. J. McCormack, Open Court Co., Chicago, 1893.

E. H. Moore, *The Foundations of Geometry,* Trans. Am. Math. Society, Vol. 3, 1902.

E. H. Moore, *On the Foundations of Mathematics* (Presidential Address), Bull. Am. Math. Society, Vol. 9, No. 8, May, 1903.

In the perusal of these books and papers, the earnest reader will find many references to other literature, which will prove of great interest, especially if he has a reading knowledge of foreign languages.

But this is merely one point of contact; and there are many others of equal interest, and of equal importance to the teacher in avoiding the installa-

tion of what I might call *false truth* into young minds—truths which require much explanation to make them convey a truthful impression; falsehoods disguised in a cloak of intuitive fallacy.

The question "What is a curve?" is, for example, a much more interesting one than appears on the surface. It is extremely easy to see that a curve cannot, in general, be regarded as the "path of a moving point". While it is best to give some such definition to the young student, it is surely advisable for the teacher, if possible, to familiarize himself with the demonstrated facts, of which I may mention a few:*

(1°) *There are continuous smooth curves which have no tangents at any points; such curves cannot be generated by motion as ordinarily conceived.* It is perhaps even more remarkable that curves exist which *have* tangents at an infinite number of points in any portion, no matter how small, but which *lack* tangents at an infinite number of other points in the same portion. But such curves may have a length.

(2°) *Curves exist which have tangents at every point, and which have no corners, but which have no curvature, no radii of curvature, and no centers of curvature, in any sense whatever.*

(3°) *There are curves which are continuous at an infinite number of points in any portion, no matter how small, but which are discontinuous at an infinite number of other points in the same portion.* Such curves, if plotted in the ordinary manner, would appear to be perfectly smooth, in many cases.

(4°) *There are curves which one can cross without cutting.*

(5°) *There are continuous curves which entirely fill an area.* For that matter, it would be equally easy to construct a curve which entirely fills a portion of space.

This list is not, of course, intended to be complete in any sense, but is merely intended to give a rough idea of the remarkable results to be found in this field. The teacher who feels interested will find an abundance of literature on the subject, in the recent volumes of the journals which have been mentioned; and many references to foreign books and periodicals will be found in these papers themselves.

Again, in the definition of such words as *"motion"*, *"congruence"*, and *"length,"* the most careful consideration is necessary. That "a straight line is the shortest distance between two points", is, for example, a statement whose apparent simplicity covers a multitude of fallacies. For the young student, it is indeed best to give such definitions, faulty though they may be, and to treat motion as a preconceived idea. But it is clearly of importance to the *teacher* to make clear to himself the complex relations existing between these three concepts. Logically, it would perhaps be best to define congruence first, and then to define motion and length in terms of congruence. But whichever order is selected, care must be taken to avoid the use of one of these terms in defining another of them, before it itself has been defined. That this is not usually done, will be apparent from the most hasty examination of the definitions ordinarily given; and it is doubtless *best* to pass over the subject with more or less intuitive explanations, in pre-

*The author regrets the necessity of stating these theorems dogmatically.

senting these motions to a class of young students, especially when it is remembered that the first logically justifiable definition of length occurs in the Integral Calculus. What I have said here is then in full accord with the previous proposal to use the intuition freely in elementary instruction. The teacher, on the other hand, should not be ignorant of the fallacies involved in the ordinary argument: (a) that the straight line has no inherent property of being the *shortest* line, until we have defined what we shall mean by the word "length;" (b) that there are reasonable definitions of length, which lead to practical results of value, for which the straight line is *not* the shortest distance between two points; (c) that our notion of ordinary motion is susceptible, besides the extraordinary changes involved in the above geometries, of simple restrictions, such as the elimination of rotation, which lead to the most beautiful results; or of extension, as by the introduction of reflection, dilitation, and so forth, under the category of motions; (d) that the idea of motion may be derived logically from that of length; (e) that the axioms of congruence are at the basis of the ideas of length and motion; and (f) that a strictly logical definition of all three terms may be arrived at by means of the Integral Calculus, the length being defined by means of a definite integral. These are of course only a few of the many interesting and valuable facts. That geometries exist in which the shortest lines are straight lines, but which do not coincide with ordinary Euclidean geometry, has been known for many years, and the Cayley-Klein geometries offer the most beautiful examples of this type. What forms have been given to these developments in recent years, may be traced in the papers read before the American Mathematical Society which are reported in the Bulletin, in papers published in the periodicals already alluded to, and in papers to which references will be found in the latter.

In this connection the treatment of ratio and proportion usually given in secondary schools naturally comes into question. The teacher is recommended, if he reads German, to study the elegant and extremely simple presentation given by Mollerup, in an article in the *Mathematische Annalen* for 1902, which can be found in the University library.

Another very practical conception is that of number; and we find that many common assumptions are invalid. Is there a least number greater than zero? How shall we convince ourselves of its non-existence? Are there any two numbers, such that no multiple of the smaller exceeds the larger? Curiously enough, there might be, in the systems of axioms usually assumed for elementary arithmetic; and it is necessary to exclude this possibility by explicit statement, in the form of an axiom, if we wish to avoid such extraordinary and undesirable combinations: that this *is* advisable appears when we consider that motion, as ordinarily conceived, would be impossible in a geometry in which such quantities were allowed to occur. The avoidance of a bad use of the words "infinity", "infinitesimal", "infinitely small", and so on, becomes imperative, under the circumstances; and the teacher will surely lead the student to the most vicious misconception, unless he is very careful to make the whole matter clear in his own mind, and to choose his language at all times with the utmost care, when referring to these delicate subjects.

It should be borne in mind constantly that the above axiom, known as Archimedes' axiom, though too nice to be introduced into elementary instruction, is really at the basis of all arithmetic and geometry; and the astounding properties which an arithmetic, or a geometry, may have, if this axiom is expressly omitted or violated, should prove an effective warning against its neglect by teachers, in their expressions before students. It is not uncommon, for example, for teachers of geometry to refer to infinite quantities in such a way as to imply a direct violation of this axiom, and this before students whose minds are in the formative stage; while the references sometimes made to infinitesimal quantities by teachers and even by books on the Calculus, seem to denote an utter disregard of its fundamental character.

Is it possible to distinguish between the number of positive integers and the number of positive fractions? It is indeed singular that it is not. But we *can* distinguish between the number of positive integers and the number of positive numbers as a whole (including the incommensurable numbers). But, again, there is no means of distinguishing the number of points on a line (i. e., the whole number of numbers) from the number of points in a square, or from the whole number of points in a plane; for these two sets of points can be paired off, one to one, so that each point of the line is mated with one point of the plane, and conversely. These are, however, merely isolated questions, which are suggested only to inspire curiosity and inquiry, on the part of any to whom they may be unfamiliar. Nevertheless, precisely these questions, and other similar ones, are coming to play a rôle in modern mathematics which is fundamental, and almost preponderant.

I have tried to make clear my two first propositions: that it would be well for the teacher to familiarize himself with the results of modern mathematical research as far as possible; and that in so doing, it is not that the direct instruction to the student would or should be altered in *form,* but that the teacher's own increased appreciation might reflect indirectly upon his teaching *truer conceptions of things as they are.* I have not touched upon such important subjects as limits, and infinite series, because a study of these subjects would be of less direct influence upon elementary teaching, the mastery of their fundamental principles would involve a more thorough training than I have wishd to assume, and ignorance of them would be less likely to lead to inefficient teaching in secondary schools. On the other hand, I have laid special stress on the matter of axioms, on account of its importance, and on account of its present accessibility in a form which the average teacher can master.

In addition to the suggestions already made, I would suggest to any teacher interested in improving his knowledge or his teaching, two methods of doing so, aside from the study of books. In the first place, there might well be, on any teacher's table, a copy of some reputable periodical magizine or journal devoted to Mathematics. This should be, however, a journal in good standing, and not one devoted to the solutions of trivial problems. At least one such is, fortunately, available: the *Annals of Mathematics,* published quarterly at Harvard University, by a board of editors from different universities. Some of the articles which appear are readable without any advanced

knowledge, and others will inspire the reader to further investigation and study. This is the only American mathematical magazine in which elementary work appears, which can be generally recommended; but *it* can be recommended to any serious teacher whose training includes at least two years of college work. For the other American journals to which I have referred, as well as for foreign books and periodicals, the average teacher might well rely upon the University library.

Another means which suggests itself at once, is close alliance to the University; and this will recommend itself to the members of the Schoolmasters' Club. Attendance at the meetings of the Club, and use of the library at such times, to clear up difficulties which may have arisen in the interim; attendance at the Summer School; or, better still, if possible, an occasional whole year spent at the University, will help the teacher surely and remarkably in his work.

I will try to be as brief as possible in the discussion of my remaining theses. They regard the student rather than the teacher.

It is not a new idea that mathematical instruction should be illustrated by practical problems, drawn from Physics and from other sciences. In fact, every branch of mathematics has had its *origin* in problems of an *extremely* practical nature. But the separation which was effected in the early part of the nineteenth century, between mathematics and its applications, has been carried to such an extreme that the studies of a purely mathemathical nature and those applications pertaining to Physics, have been wholly separated, not only in the secondary schools, but also in our colleges and universities. From the standpoint of the student this state of affairs is extremely deplorable. It leaves the study of mathematics *dry, unpractical,* and often *meaningless*, to him; and, whether he is to go on in mathematical studies, pure or applied, or whether he is to leave the subject permanently, it is of equal importance that he should be made to see the bearing of the subject, and its real significance in its applications. At just this time it is worth the effort to try to effect this change, the advisability of which is generally admitted, in a greater or lesser degree. For the leading minds in mathematics, physics, and engineering are just now giving their most thoughtful attention to a similar necessity in advanced work. Poincaré, himself a mining engineer by training, is now the acknowledged leader of pure mathematics in France; and his papers in the journal "L' Enseignement", and in the Transactions of the Paris Mathematical Congress,* demonstrate conclusively the practicability of this change, and its advisability from the standpoint of the future engineer, as well as from that of the future mathematician, or, indeed, from that of the student temporarily engaged in mathematical study.

The most extreme agitation at present is in Germany and in England. In Germany, proposal after proposal each meets its counter-proposal in

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*For precise references, here and later, see Moore, l. c., Bull., Vol. 9, No. 8, May, 1903, pp. 492, etc. See also Poincaré, La Science et L'hypothése, Paris, Gauthier Villars, 1903.

schemes for the revision of elementary and college instruction in mathematics—from engineers on one side, and from mathematicians on the other. The recent files of the "Bericht der Deutschen Mathematiker-Vereinigung", and the reports of the metings of the German Association of Engineers, teem with discussions which, although tinged with some local color, cannot fail to be illuminating and helpful.

What *can* be done is shown most plainly in the work and in the writing of John Perry, Professor of Mechanics and Mathematics at the Royal College of Science in London, and Chairman of the Board of Examiners of the Board of Education, in mathematical subjects. His work in his official capacities, and in connection with several committee reports on the improvement of mathematical teaching, as well as his actual experience in teaching mathematics of no mean order, to mechanics and apprentices in the London night schools, and still more his books and papers resulting from these experiences, are highly suggestive and supremely original.* Especially does his "Calculus for Engineers" show possibilities in such a subject for wide divergence from present methods of presentation, with reasonable hope for great improvement. While we cannot unhesitatingly approve all that Perry does or says, and while such changes as are to be made, must be carried out slowly in order to be permanent, yet his ideas may certainly be studied to advantage, and they are fortunately accessible in his papers, many of which are to be found in the library.

My latter theses are so ably maintained by Professor Moore in his Presidential Address, that I would best repeat his very words, were I to enter upon a detailed discussion of them. In fact, to sum up the results of his reasoning, we would be led to practically restate these propositions, with considerable amplification and appropriate explanation.

Let me attempt to sketch such a resumé. In the first place, that elementary mathematical instruction should find its logical explanation in examples drawn from practical physics and mechanics. There seems to be no doubt that this is actually feasible at present, if teachers and authors of text-books would unite to secure the desired end. That Physics and Mathematics should eventually be consolidated, is one of the propositions advanced by Professor Moore; and, while we cannot hope for this result immediately, it does seem possible and advisable that much of the matter and some of the examples now included in High-school and Freshman college Physics, should be incorporated in text-books and in courses in High-school mathematics. At least the student should be relieved of the idea that truth in one room, under one instructor, is essentially different from related truth taught across the hall, by another; *such* is, and has been, the extreme divorce of Mathematics from Physics in our High-schools, and of Mathematics from Physics and Engineering in our colleges and universities.

It has been suggested that such a reform would necessitate the omission of some of the material now given; and it is doubtless true that such a pro-

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*Again I would refer the reader to the bibliography given by Professor Moore, in footnotes to his Address.

gramme as that proposed by Professor Moore, if carried out in full, would cause some abridgements. It would seem, however, that such sacrifices would be more than compensated for by the improvement in the students's general grasp of the subject, and that a net gain could be realized. It is very doubtful to my mind, for instance, whether the student gains much from the *logical* part of *solid* geometry. The logical forms have all occurred in the teaching of *plane* geometry, and hence the processes used in the logical deduction of theorems are practically, for educational purposes, a needless repetition. If *plane* geometry could be assigned more time, at the expense of *solid* geometry, so as to afford time for the introduction, into plane geometry, of a few physical problems, and of some of the laboratory work of which I shall speak, I believe the student would *learn more geometry* in the end. Of course I would not neglect that great merit of solid geometry—its development of the space intuition, and of the ability to project a plane diagram into a space figure, and conversely. But I believe that the teaching of solid geometry could be *stripped down to almost nothing else,* by assuming most of its theorems by an appeal to the intuition, which needs practice if not development, and by emphasizing what is *new* in solid geometry, *only*. The general notions of projection and the general space relations, can be taught without the accompaniment of the petty logical processes which usually crowd the first part of a solid geometry text. I would propose even more thorough drill in these things, with even additional matter, taken from the elements of descriptive geometry and of the theory of perspective, accompanied by the drawing of diagrams, and the construction of actual models. With this, much attention should be paid to such new and important general principles as that of Cavalieri; which, as far as I know, is to be found in only one American High-school text—that of Professors Beman and Smith.

The same general remarks apply to the teaching of Algebra. Doubtless some of the matter now taught could be sacrificed, if necessary. In particular, the treatments of series, permutations and combinations, simple and compound interest, and so on, might be omitted. If we could introduce, in their stead, algebraic problems drawn from actual problems of physics, mechanics, and geometry, a gain might be effected in the student's total algebraic power. And now that the cross-section paper of analytic geometry is in common use for diagrams in mechanical machines of all sorts, by railroad offices, by insurance men, by even the daily newspaper, it seems certain that a high school student could grasp the use of this convenient mode of expression, as applied to practical uses and to practical problems, if not as a graphical means of studying equations and their roots. Perry's essay on "Square Paper" would be helpful, in such a connection, in freeing analytic geometry from its incubus of artificial names, words, and signs.

What I have said naturally leads to the idea of a mathematical laboratory, for the suggestions already made would be quite sufficient for a beginning. What with the drawing of accurate figures, and diagrams of solids, supplemented by some practice in practical draughting; with the construction of actual models in paper, wood, tin, plaster, and other available material; with the metrical experiments of weighing and otherwise measuring solids

and planes; with the construction of graphical time-tables, physical laws, statistical curves, and other similar figures, on cross-section paper; all the time and means at present obtainable for the purpose, could be profitably spent. I believe that the outlook for a mathematical laboratory in the high school is at present actually brighter than was the outlook for a high school physical laboratory, twenty years ago, not to speak of chemical, biological, and other laboratories, which are in common, and highly successful, operation in our High-schools today. What the development of the High-school mathematical laboratory might be, and how beneficial in its effect on the student's grasp of the subject, may be guessed from the known results in other subjects, if it is not inherently apparent.

The same general ideas apply, with added force, in fact, to instruction in colleges. Illustrations and applications drawn from other sciences are, indeed, not wholly lacking, but they might be made vastly more effective and helpful, by increasing their number and their practicalness. Especially is this true of the Calculus, which so sadly often leaves the student with the notion that mathematics is a dreamer's raving. Might not more time be given to the applications of the Calculus to Mechanics, and less to the traditional "applications" to the theory of curves and surfaces? Is not the idea of a derivative more clearly illustrated by a *velocity* than by the slope of a curve?—at least it was in this way that Newton conceived of it, when he invented the Calculus. And can there be any comparison between *acceleration* and *curvature* as a simple illustration of a second derivative?

But it was not my purpose to touch upon the instruction of mathematics in colleges, so much as to urge reform in our secondary institutions. As for the laboratory idea, I will merely mention that modest laboratories have already been started at Yale and at Chicago, at least. Let us hope for their success and development, and for the gradual introduction of laboratories in all large colleges.

Finally, it is apparent that such changes cannot be made by teachers in secondary schools without the co-operation and assistance of the University. Texts must be written, the use of which is at least not fatal to such a course; but these will come in time; our recent text-books show enormous improvements in precisely these directions. Teachers must be trained to understand the meaning and the purpose of the proposed reform; but our present university instruction is sufficiently literal to give hope that a beginning might be made at once. The only serious difficulty in the way of an immediate beginning is the obvious necessity of a change in college entrance requirements.* It is clear that an alternate system of examinations would be needed, which would admit of either method of instruction. But this change *must be made at once,* if any considerable alteration in our High-school Curriculum is to be fostered. That such a change would be advisable is not the isolated

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*In this connection mention should be made of the recent Provisional Report of the Committee of the American Mathematical Society on Definitions of College Entrance Requirements in Mathematics, which has been sent to the members of the Society.

opinion of the writer, but is maintained by all those who look for improvement in secondary instruction along the lines which I have suggested; and I trust that those in authority in all of our larger colleges, will see their way to a *gradual* change in this direction. It is particularly desirable in the State of Michigan, that the stimulus for improvement should come from the University, on account of the ascendancy of the University over the secondary schools of the state, and on account of the intimate relations existing between the University and the Schoolmasters' Club. Should the members of the Club coöperate with the University authorities to promote and encourage such reforms as those which I have indicated, I believe that a marked improvement would be felt immediately in the instruction in mathematics, throughout the secondary schools and the colleges of Michigan.

I would reiterate, however, that my statements are not intended to be a final *dictum* on the subject. But that some such reforms are inevitable, and that we must either bend or break, under the pressure of their necessity, seems certain. I shall feel highly gratified if my tentative remarks only lead to serious discussion, and to awakened interest on the part of the members of the Club.

MATHEMATICS IN THE SECONDARY SCHOOLS OF NEW YORK CITY.

CLIFFORD B. UPTON, HORACE MANN HIGH SCHOOL, NEW YORK CITY.

A visitor to the busy metropolis who expects to find a thoroughness and enthusiasm in the teaching in the secondary schools which corresponds to that in the commercial life of the city will not be disappointed. He will discover school work here which is in all respects equal if not superior to that of any other city in the Union, and will leave with the feeling that much of value has been gained from his observations.

Greater New York, which is a single municipality formed by the consolidation of New York, Brooklyn, Long Island City, and several adjoining towns, has in all twenty public high schools which have a combined attendance of eighteen thousand students, about two-thirds of the total number attending all the high schools in Michigan; five thousand more students are found in the private secondary schools, all of which do excellent work and which also must be visited if one would become thoroughly familiar with the teaching in the city. It is readily seen that it requires no small amount of time to visit but a single department of so large a number of schools, yet an examination of the typical schools will give one a just estimate of the entire system. Having spent one week visiting the mathematical departments of the most prominent public and private high schools in New York and Brooklyn the writer will attempt to give some idea of what is being done in them in the teaching of secondary mathematics.

NEW YORK CITY

Year of Course	Horace Mann High School (Private)	New York High Schools (Public)	New York Commercial High School (Public)	Dr. Sach's Collegiate Institute (Private)	Chicago High Schools (Public)
I 1st sem.	alg. 5	alg. 5	alg. 4	alg. 2	alg. 4
I 2nd sem.	alg. 5	alg. 5	alg. 4	alg. 2	alg. 4
II 1st sem.	pl. geom. 5	pl. geom. 4	pl. geom. 3	pl. geom. 2 alg. 3	pl. geom. 4
II 2nd sem.	pl. geom. 5	pl. geom. 4	pl. geom. 3	pl. geom. 2 alg. 3	pl. geom. 4
III	2 1 *alg.-geom.* 3	pl. geom. 3	pl. geom. 3	pl. geom. 3 alg. 3	*alg.* 4
	2 1 *alg.-geom.* 3	alg. 3	alg. 3	pl. geom. 3 alg. 3	*sol. geom.* 4
IV	*trig.* 3	*sol. geom.* 4	*sol. geom.* 4	pl. geom. 3 alg. 3	
	sol. geom. } *alg.* } 3	*alg.* or } *trig.* } 4	*trig.* 4	pl. geom. 3 alg. 3	*trig.* 4
	32	32	28	38	28

The numbers after each subject indicate the number of recitation periods a week.

The length of the recitation period in each of the above New York courses is 40 minutes.

The numbers under each course indicate the number of recitation periods per week for one semester which would be equivalent to the total time spent reciting mathematics during the four years.

Subjects in italic are elective.

The course of the Chicago High Schools is added for comparison.

The course of study in mathematics first attracts our attention. It is the same for all the twenty public high schools of greater New York, except the manual-training and commercial high schools, and even in these the variation from the regular course is very slight. Furthermore, equal amounts of algebra and geometry are covered in the various schools during each semester, the exact requirements being given in a syllabus recently prepared by the superintendents, after consultation with the heads of the mathematical departments. Referring to the tabulated course of study it will be noticed that plane geometry occupies all of the second year and the first semester of the third, while solid geometry is given the first semester of the fourth. The syllabus just mentioned requires in the second year the first four books of plane geometry plus a minimum of three hundred exercises, and in the first half of the third year the completion of the subject plus a minimum of seventy-five exercises. In algebra simultaneous quadratics must be finished in the first year, while in the third year the first year's work is reviewed, and ratio, proportion, series, and the binomial theorem are added. In the mathematics of the fourth year the student is prepared for his college entrance examinations and is given practically what he needs for the particular college he wishes to enter; he may have solid geometry and plane

trigonometry or even advanced algebra, if he happens to be preparing for some technical school. The public high schools can easily satisfy the various demands in the senior year, as each large high school has from ten to fifteen teachers of mathematics, with each course divided into several sections.

The course of the Horace Mann School is a typical private school course. Plane geometry is finished in the second year, and is reviewed one hour a week during the third year, in connection with a two-hour course in algebra; the fourth year is similar to that of the public high schools. The other courses explain themselves. Dr. Sach's school is especially strong in mathematics, and the course is of interest in showing what division is made to carry algebra and geometry side by side during the last three years. A general examination of all the New York courses will now show the following important facts:

(1) There is an almost absolute uniformity in all the courses of both public and private schools.

(2) The work is extended over the entire four years, so that no semester is without its mathematics. The work is thus constantly kept fresh in the student's mind, and this without any disturbance in the sequence of the course.

(3) A requirement of three years work in mathematics from all students in the public high schools.

PUBLIC HIGH SCHOOLS OF MICHIGAN

Year of Course	Detroit	Jackson	West Saginaw	Ann Arbor	Kalamazoo	East Saginaw*
I	alg. 5 alg. 5	alg. 5 alg. 5	alg. 5 alg. 5	alg. 5	alg. 4 alg. 5	*alg.* 4 *alg.* 4
II	alg. 5		alg. 5	alg. 5 alg. 5	alg. 5	2 3 *alg-geom.* 5 2 3 *alg-geom.* 5
III	pl. geom. 5 pl. geom. 5	pl. geom. 5 pl. geom. 5	alg. 2½ pl. geom. 5 pl. geom. 5		geom. 5 pl. & sol. geom. 5	*geom.* 2 *geom.* 2
IV	*sol. geom.* 2½ *trig.* 4 *alg.* 2½	*sol. geom.* 5 *trig.* 5 *alg.* 5	sol. geom. 2½	geom. 5 pl. & sol. geom. 5 *trig.* 5	*trig.* 4	*alg-geom.* 3 *trig.* 2 *alg-geom.* 3 *trig.* 2
	34	35	30	30	28	32

*East Saginaw has required work in algebra in the seventh and eighth grades.

Subjects in italic are elective.

For the sake of comparison several of the typical Michigan courses have been added. No longer can it be said that the chief difference between the courses of study in the East and the West is that the East devotes one year

to plane geometry while the West devotes one year to both plane and solid; the East has added solid geometry, although as an elective, and the West is giving more time to plane. Detroit, Jackson, and West Saginaw each devote the entire third year to plane geometry, putting solid geometry in the fourth, while Kalamazoo and Ann Arbor still cover both plane and solid geometry in a single year. This leads us to make a general examination of the Michigan courses with these results:

(1) There is no general uniformity in the courses, although Detroit, Jackson, and West Saginaw are quite alike.

(2) There is no school which spreads the work over the entire four years, except East Saginaw. In that city this result is attained by dividing the subjects to such an extent that a student changing from one school to another before graduation would be placed at a considerable disadvantage.

The numbers under each course of study indicate the number of recitation periods per week for one semester, which would be equivalent to the total amount of time spent reciting mathematics during the four years. It is thus seen that some of the Michigan schools spend more time upon mathematics than either the New York public schools or the Horace Mann School, and still do not secure the New York advantages of continued sequence and uniformity. The course of the Commercial High School of New York is an excellent example of what may be done in the distribution of the work over the course, using no more time than the minimum of any Michigan school.

The extension of the work through each semester of the course is secured by placing plane geometry in the second year. This plan is so simple as to raise the question as to whether there is any reason for not placing it there. By starting slowly, and devoting a year or even more to plane geometry, a student can take it up in the second year with an interest very much greater than that with which he would study algebra in that year, and in addition the plan gives him a good general mathematical training before commencing the study of physics. This would leave the third and fourth years for an arrangement similar to that in the New York courses.

With much interest the writer has watched the courses in Michigan for several years and it certainly seems that they are in a stage of transition. From a single year's work in plane and solid geometry in the fourth year they are changing to a whole year of plane gometry in the third year and a semester of solid geometry in the fourth, with a general tendency to extend the work over each semester of the entire four years. Uniformity and division seem to be the goal, and the writer firmly believes that a few more years will find all of the Michigan courses uniform with those of New York and Chicago.

The teachers of the secondary schools in New York are an interesting body; their scholarship is high, their experience large, and their personalities are especially attractive. There are three grades of teachers in the public high schools, and before election a teacher must possess the qualifications of the grade for which he applies in addition to passing a very rigid

examination in mathematics, the methods of teaching it, and the science of education. The requirements of the different grades are as follows:

The head of the department, called first asistant, must be a graduate of a recognized college or university, and must have had at least one year's satisfactory post-graduate work, besides five to seven years of successful experience in teaching in secondary schools or colleges. His salary for the first year is $2,500 with an annual increase of $100 until a maximum of $3,000 is received.

The second assistant must be a college graduate and have had at least two years of successful experience in teaching in secondary schools or colleges. The first year's salary is $1,300 with an annual increase of $110 until a maximum of $2,400 is received.

A teacher of the junior grade must have a college degree and have completed a satisfactory pedagogical course of at least a year, all in addition, of course, to the above mentioned examinations. The salary starts at $900 with an annual increase of $50 until a maximum of $1,200 is reached. All the salaries quoted are those of men, the women receiving salaries correspondingly high, though a little less in amount.

What is the result of these requirements and these salaries? There can be but one result and we find that here. The teachers are thoroughly prepared for their work, and, in addition, are broad-minded, liberally cultivated men and women. They are teachers in every sense of the word. In the private schools the faculties are equally competent, and men with master's and doctor's degrees are found in almost every such institution.

It is of interest to note in this connection that New York City now has the best facilities of any city in the country for the thorough preparation of teachers of mathematics; more extensive courses in the pedagogy and history of mathematics are offered than in any other city, while certain collections of original sources here, which bear directly upon the teaching and history of mathematics, are now the largest and most complete in the world.

As to the spirit of the work and methods of teaching much can be said. It is just the spirit that one would expect from teachers with the education, training, and personality required by the New York Board of Education.

First of all, thorough, rigid mathematics is taught. Every detail is thoroughly emphasized and the student is expected to master it. There is an excellent appreciation of a student's needs, while mistakes and difficulties are skillfully cleared away. There is a constant requirement of clear thinking, with exact and concise statement, while carelessness is not tolerated for an instant. But this emphasis of the long recognized disciplinary features of mathematics is not all that is found; the distinctively newer life and scientific spirit which have now come into the teaching of secondary mathematics are discovered to quite an extent, and give evidence of spreading very widely throughout the city within a short time.

In algebra the syllabus in mathematics just prepared for the public high schools, advises the teaching of detached coefficients, the remainder theorem, the treatment of quadratic equations immediately after factoring, and the checking by short methods of not only all the fundamental opera-

tions and factoring, but also of the roots of all equations. Many of the teachers have been doing these things for some time and have also used graphs very freely in the study of equations.

In geometry it is gratifying to see the interest taken in exercises, the syllabus requiring that at least 75 be taken up with each book of geometry, and some of the contributors to the syllabus even favored requirements and suggestions as to methods of attack for these exercises, although this feature has not as yet been inserted. However, the fact exists that many of the teachers have already made a good beginning in presenting general methods of attack, and an extension and requirement of this practice is soon looked for.

The writer found several teachers who also introduced choice bits of the history of mathematics to make the work more real and interesting.

So anxious are a few of the teachers to improve their work, that they are trying the no-text plan in geometry and the individual or laboratory method for algebra, and, it is claimed, with excellent results. The great majority, however, are following the plan which long experience has proved the best, i. e., of following a good text and carrying on a live recitation.

Another commendable thing is that there is no uniform text which every school must use; the Board of Education furnishes a list of 42 of the best American algebras and 40 geometries, from which each high school may select the texts it wishes. It is also of interest to add that several of the most prominent private schools have lately discarded the older texts for the more rigid and modern books.

The above are not theoretical statements but are the results of the writer's personal investigations in the schools themselves.

So far the general spirit of the work in the public high schools has been discussed. A few words may now be in place concerning the work in the largest and most prominent private school in the city, the Horace Mann School. This is the school of observation of Teachers College of Columbia University; it is supported and controlled by Teachers College and is a part of the Columbia University system. It has an attendance of a thousand students, about one-half of whom are in the High School. It is a private day-school, and a high tuition is charged, but it is not a school for general practice teaching, Teachers College maintaining a free school in another part of the city for that purpose.

The mathematical department of the high school is supervised by Dr. David Eugene Smith, professor of mathematics in Teachers College, and it is in this high school department that the spirit and methods of the pedagogical courses given by him in Teachers College are carried out. The spirit of the teaching is entirely modern, and all of the real improvements in the teaching of mathematics are here put into practice. It is not a department of fads but a place where methods of teaching are used which the history and pedagogy of mathematics have proved to be thoroughly sound.

All of the rigorous teaching noted in the public high schools is here found, and in addition all of the usable modern notions are taught with great success.

In algebra the students are given almost their first ideas of formal mathematical reasoning, and find that the subject is made up of definite principles. They better appreciate the meaning of the new kinds of number in algebra because they have been given parts of the history of our interesting number system. When an expression to be factored does not fall under one of the ordinary types, they quickly and intelligently apply the remainder theorem and they know their results are correct because their work checks, an answer book being an unheard of thing; while the usual maze of quadratic equations is clearly seen to be nothing but factoring. It does not require an extra amount of time for this modern work, but on the other hand, it is found that the interest in it rather facilitates than retards the speed over the required ground. This work is all done in a thirty-three weeks' school year by students of only average ability who all carry heavier programs than the average public high school student.

In geometry a like interest is found, and this too in the second year of the high school course where experience has shown that geometry can readily be taken up. Principles of wide application are readily grasped, such as those of continuity and the law of converse, and the students delight in generalizing a figure or in proving a converse theorem by means of the law. General methods of attack are early given to the student, and the exercise work is full of enthusiasm as a result. The history of geometry is frequently found to be a means of adding intense interest to the work, the boy knows why in elementary geometry he uses only the straight edge and compasses, and why it is useless for him to attempt to trisect an angle. Here, too, it is found that the work is made lighter rather than heavier by an appeal to the modern methods.

Another source of inspiration to the students is the high school mathematical library. Here are found books on mathematical recreations, the best histories of mathematics in English, the classic texts in algebra and geometry, besides many little books, like "Flatland," which make one feel that mathematics is not only a very interesting but a really fascinating subject.

The department of mathematics also possesses a complete set of the latest German models for illustrating the theorems in solid geometry. These are, however, used with considerable discretion, and are not allowed to rob the subject of its value as an exercise for the imagination.

The teachers are enthusiastic and get their enthusiasm from the head of the department and from their acquaintance with the very best literature upon the subject. All the leading foreign and American texts are available and are consulted. The work is thoroughly systematized and there is a general uniformity in method and spirit throughout the department.

In conclusion it can be said that the teaching of secondary mathematics is exceptionally good in New York City and this excellence is not accidental, but the result of natural causes. A good course of study, and salaries sufficiently high to obtain thoroughly prepared teachers must produce good results. And further, with the awakening to the fact that not only a thorough academic training, but also a strong course in the pedagogy of mathematics is the necessary training for a teacher of high school mathematics we shall find our teaching taking on a new life.

TO WHAT EXTENT SHOULD ARITHMETIC, ALGEBRA, AND GEOMETRY BE TAUGHT TOGETHER?

ARTHUR G. HALL, PH. D., UNIVERSITY OF MICHIGAN.

A text for this paper may be found in one of the recommendations of the mathematical conference of the Committee of Ten, viz., "The course in arithmetic should be at the same time abridged and enriched." Deferring the question of abridgement, the more immediate topic will be the enrichment of the course in accordance with the demands of the subject-matter of the great science of mathematics and of the nature and development of the child.

It would be interesting to trace the origin of the present arrangement of the mathematical studies of our primary and secondary schools,—first for eight years repulsively and fatally large portions of arithmetic in the American acceptation of the word, then for a certain period algebra as a study entirely distinct from the preceding, then demonstrational geometry quite as distinct, each isolated from and neither helping nor helped by the others. Was it due to the limited attainments of the instructors in those branches? Or was it in obedience to the commercial spirit of the communities that the mathematical portion of the course pursued by the majority of children should be devoted to those topics popularly supposed to be helpful in business transactions, but now admitted to be of very slight value either commercially or mathematically? They not only fail to bring about that quickness and accuracy in computation essential to success in either business or student life, but effectually kill off any fondness for mathematical study or powers of clear and exact reasoning.

It is obvious that the foregoing classification is highly artificial. The algebra of our secondary schools is esentially the same as the arithmetic which goes before, differing only in the generality due to the employment of the literal notation and the introduction early in the process of solution a symbol representing the unknown desired number. A more scientific classification would seem to be the following: Arithmetic, Geometry, and beyond these Analysis. Arithmetic is the science—and, like all the others, the art as well—of constant numbers, discrete at first, later also continuous number. It includes the elementary arithmetic of the first four fundamental operations, the literal or general arithmetic commonly called elementary algebra, and higher than these the theory of numbers and the theory of number systems. Geometry concerns itself with form and space relations and includes observational, constructive, demonstrational Euclidean and non-Euclidean, projective, vector, and analytic or coördinate geometry. The term Analysis is used to include all that nongeometric mathematics which results from the introduction of continuous variable number, beginning with the calculus, the threshold to all higher mathematics.

In no branch of higher mathematics do we find ourselves concerned exclusively with one of the two latter classes, but continually use processes and illustrations taken from both with the utmost freedom. The form side and the number side are but the two aspects of the one science of mathematics. This mutual dependence and helpfulness are equally characteristic of the elementary branches. Arithmetic in its beginnings has to do not with abstract but with concrete numbers. It grows by means of and concerns itself very naturally with geometric relations and measurements. On the other hand geometry at once depends on and enriches the arithmetical and algebraical processes and results, while elementary algebra is, as I have said, a generalization of the other two and affords an invaluable mechanism to aid in their investigation.

Not only is all this true from the standpoint of the nature of the subjects themselves. It accords equally with the needs of the developing mind. If we assign to either arithmetic or geometry the earlier place in the growing child's mind, it must be to the latter. Forms of objects constitute one of the first and most fruitful and abiding concerns of the child's mental activities. He learns to count as he learns to distinguish between different objects, and the relations between the parts of objects afford an easier and more interesting occupation than the relations and combinations of abstract numbers. Even when the child has advanced from the observational to the reasoning stage, how many high school seniors can either comprehend or give the demonstration of a theorem in algebra as understandingly and successfully as that of one in geometry?

In view of this intimate interrelation between arithmetic and geometry, and because they form but two aspects of the science and art of mathematics, it follows that they should be developed side by side in a course of instruction pretending to educate the whole child. This should, of course, be done with due regard to the peculiar rights and values of each. For example, danger lurks in the too complete substitution of the numerical method of demonstration for that which is purely geometric: The Pythogorean proposition, for instance, may be demonstrated by a purely geometric method or by a substitution of the symbols representing the number of units of length in the sides and the employment of the algebraic theorem of the square of a binominal. In the report of the Committee of Fifteen, in an argument against the early study of geometry, the one method is mistaken for the other.

Concrete or observational geometry ought then to be given a place early in the curriculum and throughout. The chief ends to be secured by early geometrical teaching are accuracy of observation and definiteness of imagination. These uses of geometry have been strangely neglected by both friends and foes of this intellectual training, who have considered only the continuity of thought and the consecutive dependent argument. But the powers of perception and of imagination, or conception, are of quite as great importance as those of reasoning. They give us the basis of facts without which reasoning is vague and worthless. The early study of the child, therefore,

while seeking to develop his ideas of space and time, must do so by training him to rapid and exact observation, clear and definite conception.

This observational or inductive side of mathematics may well be emphasized. It is common for non-mathematical writers to consider all mathematics as of the same nature as Euclid's Elements, starting from certain assumed definitions and axioms and proceeding by connected deductive reasoning to a final conclusion. Says Professor Huxley: "Mathematical training is almost purely deductive. The mathematician starts with a few simple propositions, the proof of which is so obvious that they are called self-evident, (sic!) and the rest of his work consists of subtle deductions from them." And again: "Mathematics is that study which knows nothing of observation, nothing of experiment, nothing of induction, nothing of causation."

Lagrange, than whom no higher authority could be quoted, has expressed emphatically his belief in the importance to the mathematician of the faculty of observation. Gauss has called mathematics a science of the eye. Riemann has written a thesis to show that the basis of our conception of space is purely empirical and our knowledge of its laws the result of observation. Sylvester, in an address before the British Association, cites many notable examples of analysis discovered by observation and established by induction and verification, the deductive proof following long years afterwards.

From this argument we must not conclude that the mathematical laws should be allowed to remain standing on an empirical basis, nor infer that mathematics affords the best training in observation and experiment. The biological and physical sciences are undoubtedly much better adapted to the latter purpose, while the inductive stage of mathematical study must gradually pass over into a demonstrational one, increasing in rigor as the mind gains in power of connected, exact reasoning. The present contention is that this gradual development should continually extend over both the arithmetical and geometrical sides of the science, so that both may keep pace with the other mental activities of the child to afford an even well rounded development. Such a course does not consist in an attempt to memorize or discover independently demonstrations for the theorems of Euclidean geometry, nor is it satisfied by including in our arithmetics numerical exercises in mensuration. The first would be time and endeavor worse than wasted in the lower grades. The latter affords a valuable enrichment of arithmetic, but little more. The study of the properties and relations of geometric forms by means of close observation, oral and written descriptions, and constructions will afford abundant material for the pursuit of geometry for itself. Time for it can well be spared from that now devoted to arithmetic, without sacrificing the true interests of that subject.

As to elementary algebra, i. e., literal arithmetic and the use of the equation and the symbol for the unknown quantity, abundant appropriate and valuable opportunity is afforded for its introduction whenever the pupil evinces sufficient ability to generalize. Thus the laws of addition, subtraction, multiplication and division and the exceedingly simple relations of

percentage all find convenient expression as formulas or equations. Here great care must be exercised that not the empty symbol but the content should ever be present in the pupil's mind. Frequent exercises in interpreting the formulas and giving complete verbal statements for them will tend to counteract this. Again the pupil must always realize that an equation is merely the symbolic writing of a sentence, a proposition in the logical syllogism of the argument.

The two likely errors in the introduction of geometry and algebra into the grades are, of course, on the one hand, a namby-pamby wasting of time, and on the other hand, an attempt to force the too immature minds to mental processes that are beyond them. To carry out this double program should not require more time than is now given to arithmetic in the primary schools. That the present results are lamentably incommensurate with the exertion is commonly admitted, and time is far too precious to waste. Deliberateness, however, not feverish haste and overcrowding, will accomplish most in the least time.

It has been the fortune of the writer to observe the results of two applications of curricula similar to that just described, both of which had passed beyond the stage of experiment into that of established success. In the German Gymnasium arithmetic (both decimal and literal) is taught simultaneously with geometry throughout the grades corresponding to those of our grammar and high schools, and about one year is gained over the best schools of our central states. For a very interesting and discriminating account of the Prussian schools with special reference to the work in mathematics, the reader is referred to Professor Young's little book.

Again in La Porte, Ind., under the superintendency of Dr. W. N. Hailmann, geometry and arithmetic were to be found throughout the first eight grades, geometry and algebra throughout the remaining four. The pupils came to the ninth (or first high school) grade thoroughly familiar with those properties of geometric forms open to observation and excellently prepared to take up demonstrational work, which was the more easily comprehended that it dealt with familiar material.

The abridgement by one year of the school course from the beginning up to the attainment of the baccalaureate is in the air. Can it be effected without sacrificing any valuable results now accomplished, say by wise revision of the curriculum and increased efficiency of methods of instruction? This is a question which, in view of the prominence of mathematics in the course of study, must vitally concern the teachers of mathematics. May not its answer involve some consideration of the changes suggested in this paper?

PHYSICS SECTION

THE USE OF MODELS IN SECONDARY INSTRUCTION IN PHYSICS.

PROFESSOR E. A. STRONG, MICHIGAN STATE NORMAL COLLEGE.

This topic is not quite in line with the usual,—and I think we would all say,—the best work of this section of the Club. It is offered as a suggestion of an occasional day upon some limited portion of our work, either of material or instruction. I have at present in mind two subjects to which my attention has been strongly called of late:—the relation of the mathematical to the physical instruction in the high school, and an exhibition of models which have been found useful in teaching physics

The subject of models is a large subject and shares in the fundamental difficulties of definition and classification common to all forms of apparatus. For this reason I shall entirely omit definition and attempt only a rough classification of models used in instruction.

CLASSES EXCLUDED FROM CONSIDERATION.

In the natural history sciences a model may mean a representation, often on an enlarged scale, of the whole or some part of some plant or animal. Large cabinets of such models in glass, wax, or other material, may be had. These are not for us.

Collections of models of machinery used in some technological process are often seen:—as, models of a spinning jenny, of a sugar mill, of an ore crusher, etc. Moderately useful as loan collections they are in general hardly worthy of purchase.

The term model is often applied to a simplified form of some complicated research apparatus; as, "Maxwell's Color-Box, Small Model", etc. May be important or not.

MODELS OF INFERIOR UTILITY.

A so-called diagrammatic model is likely to be inferior, other things being equal, to a working model. Such a model attempts to show by parts moving in one plane the action of a mechanism of three diamensions; or, if in three dimensions, it is not "practical", as the theatrical people say; or, it may represent to the eye and touch concealed or invisible parts. A representation in one plane of a section of a lifting pump, of a steam boiler,

of a Bramah press; or, in three planes, of a ray of common light, or, of the parts of an induction coil, are examples of this class. Reusch's apparatus for refraction; Pfaundler's of the course of a ray of light through compensating prisms; or Neu's for the course of a ray through a lens, in all of which the so-called rays of light are represented by wooden rods, furnish examples of this class of models. While these diagrammatic models are in general inferior in value, some of them are of high value, as, for instance, a representation in one plane of a steam cylinder with valve action and related parts. So this classification is not of itself sufficient.

A model of a complicated piece on so small a scale as to be a mere toy can have little value in class instruction.

Models showing mere theory one can in general well do without. The inventor finds enormous interest in making them, but his single example may well suffice. Models showing the nature of inertia, the interaction betwen the molecules of common matter and electricity, the nature of geyser action, of Saturn's rings, furnish examples of this class.

Models illustrating some fact or principle which needs no illustration are quite too common. For example, the rather common orrery or planetarium. I quite agree with Poggendorff that this piece is not only useless but actually harmful. The solar system certainly challenges imitation as well as admiration, and I do not wonder that attempts to represent these complicated motions and relations are often made, but why anybody should buy them is the mystery.

The paper was illustrated by models of a marine engine, of a vernier reading to hundredths, a vernier reading to thousandths, and one showing the locus of a connecting rod.

THE ELECTRON THEORY.

PROFESSOR K. E. GUTHE, UNIVERSITY OF MICHIGAN.

The electron theory which has lately attracted so much attention among scientists may be called the third stage of the development of our theories concerning phenomena connected with electro-magnetic action.

At the beginning of the second half of the nineteenth century *Wilhelm Weber* explained the electromagnetic phenomena known at that time, perfectly satisfactorily, by assuming the existence of two different kinds of electrical fluids, *i. e.*, positive and negative electricity. Electricity could be divided into small particles, atoms of electricity Weber calls them, which act upon one another through space, these actions being transmitted with infinite velocity. The idea of the electrical atom, however, did not at that time lead to attempts to measure its size. Many of our definitions and modes of speaking of electrical phenomena date back to Weber's theory.

It was *Faraday's* master mind which pointed out the short-comings of this theory and emphasized the great role which the medium plays in all electrical phenomena. Then *Maxwell* in his great treatise gave the mathematical basis for the electromagnetic theory of light, and *Hertz* by his famous researches proved that electromagnetic action is not an action at a distance but transmitted by the ether. The possibility of treating the phenomena of light and those of electromagnetic waves from the same point of view, led physicists to abandon Weber's theory and look to the ether and the lines of force, *i. e.,* the stresses produced in the ether, for the solution of all future problems in electromagnetism. According to Maxwell's theory there are no atoms of electricity. We have a continuous ether with certain electrical and magnetic properties and these properties may be modified by the presence of matter.

Now the third period in the development of the electrical theory which may be characterized by the name of the electron theory, means a return, at least to a certain extent, to the ideas of the first, or better, a combination of the two, *i. e.,* we have to allow a right of existence as well to the atoms of electricity as to the continuous ether with its electrical and magnetic stress properties.

In fact the "electric charge" has always been a more or less definite something, which has never allowed itself to be pushed aside entirely in an important branch of electricity, namely, in electrolysis. All attempts to explain electrolytic action on the basis of ether stress have found hardly any adherents among the physicists and we are accustomed to say that in electrolytes certain small particles of matter are charged with a definite amount of either positive or negative electricity, a combination which we call an "ion." When a current goes through a cell, the electric charge is given off at the electrode. It is apparent that, at least during the short time of transference, we think of the charge as having a separate existence. The smallest quantity of electricity found in electrolysis is then that amount which is carried by a monovalent ion—How large is it? We know that one gramequivalent of an ion, say 1 gram of H, carries very nearly 96600 coulombs. The ratio of the charge in electromagnetic units to the mass of one ion of hydrogen is ten apparently 9660 or nearly 1×10^4. The number of gas molecules in 1 cm^3 at 0^0 and 760 mm/cm^2 pressure of mercury is about 4×10^{19}. As an example take hydrogen. Each molecule is made up of 2 atoms; we have therefore in each cubic centimeter about 8×10^{19} atoms. One grammolecule occupies 22350 cm^3 *i. e.,* there are 178800×10^{19} atoms in 2 grs. of H. These carry 2×96600 coulombs, so each ion has a charge

of $\dfrac{193200}{178800 \times 10^{10}} = 1.1 \times 10^{-19}$ coul. $= 1.1 \times 10^{-20}$ electromagnetic units.

Since an electromagnetic unit of quantity of electricity is 3×10^{10} larger than the corresponding electrostatic unit, *we obtain from the consideration of electrolytic phenomena for the electrostatic charge of a monavalent ion about* as 3.3×10^{-10} units.

Even in the electromagnetic theory of light where the wonderful success of Maxwell's theory seemed at first to leave no room for separate electrical charges, they began slowly to gain more and more ground. It was soon found that certain phenomena of light, e. g., radiation from incandescent vapors or the absorption of light by certain substances and especially the dispersion of light, demanded a definite structure of bodies. *Maxwell, von Helmholtz* and others assumed that the material particles emitting light of definite wave length possess one or more distinct periods of vibration and will on the other hand act like resonators for these vibration frequencies and therefore absorb their energy. Thus the material particles whose atoms carry positive and negative charges, were supposed to create by their vibrations electromagnetic disturbances of the ether.

H. A. Lorentz of the University of Leyden assumed as early as 1880 that not the material atoms of the substance but only an electric charge connected with it and having a vibration period of its own produces the electromagnetic phenomena of light. Starting from this hypothesis he was able to explain by means of a formidable structure of mathematical formulæ the apparent exceptions to Maxwell's theory. But Lorentz's greatest success was to come 16 years later; his theory, similarly to Maxwell's, had to wait for an experimental proof.

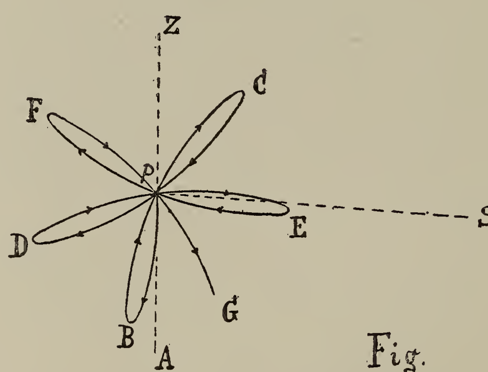


Fig.

Let us see whether his theory could predict effects of magnetic and electrostatic fields on light rays, which had not been observed before and which by being found would of course strengthen it. We suppose with Lorentz that the small electrical charges which we shall from now on call "electrons," a name first used by *Stoney,* swing rapidly about a point of rest and by their influence upon the ether produce light radiation. Let such an electron, which we consider as a negative charge, after it has left its position of rest be subjected to an elastic force pulling it back. It will swing then in a simple harmonic motion in a straight line and produce light of a definite wave length. But now suppose we bring this swinging electron into a strong magnetic field. A moving electrical charge will be acted upon as a flexible current would, *i. e.,* if we suppose the magnetic lines of force go from the front vertically through the plane of the paper and a negative electrical charge to start from the point, *P* (see figure —), vertically down parallel to

the plane of the paper, it would, instead of going to A, be deflected to the left so as to describe the path PB. After having reached its maximum elongation at B, it will return to its position but on a path concave towards the right and thus describe a loop. Constructing the path it takes during several vibrations, it will reach the points C, D, E, etc., the width of the loop depending on the strength of the magnetic field. We are accustomed to consider a motion in a loop as this as the resultant of two separate motions, say one in a vertical direction alone $A\ Z$ and one at right angles to it, along $P\ S$.

The question arises: What kind of light will be emitted from an electron moving in the manner described? Let us take the direction PS as the direction of propagation of the light. Light vibrations are always at right angles to its direction of propagation, *i. e.*, we must suppose that only the component at right angles to PS can be propagated in this direction as light, but not the component parallel to it.

It is apparent that the vibrations parallel to $A\ Z$ will alternately increase and decrease in amplitude. In sound we are perfectly familiar with such phenomena: we say we have "beats," whose effect is the same as if two vibrations with somewhat different periods were present. Our eye is not sensitive enough to see these extremely rapid light beats, but in using a spectroscope we should expect two lines of distinct wave length corresponding to these beats. Moreover according to the theory of light ether vibrations at right angles to the plane of the paper would also be propogated as light rays in the direction PS, but on these the magnetic field has no influence, *i. e.*, their amplitude remains constant. We shall therefore see in the spectroscope also a line between the other two, this line corresponding to the natural wave length of the light emitted by the body. The effect of bringing a body emitting under ordinary circumstances light of a certain wave length into a strong magnetic field and looking at it through a spectroscope in a direction at right angles to the magnetic lines of force, is therefore, that we see instead of the usual one bright line now a triplet of bright lines. This phenomena which Lorentz's theory demanded, has been found experimentally by *Zeeman* in 1896, it is the famous "Zeeman effect." Various bright lines emitted by incandescent vapours have been studied under these conditions and Zeeman has calculated from the mathematical theory the ratio e/m of the electrical charge to the mass of such a vibrating electron. For each of the two D lines he found it to be 1.6×10^7 electromagnetic units per gram, for the Cadmium line 2.4×10^7. Comparing this result with what we found above for the same ratio in electroytic phenomena, namely about 1×10^4, *we obtain the very surprising result that it is in this case more than a thousand times larger.*

Thus Zeeman by his famous discovery strengthened considerably the electron theory introduced by Lorentz into the electromagnetic theory of light.

While the new electron theory was originated in connection with Maxwell's theory, its importance was seen more clearly when purely electrical phenomena also demanded the existence of electrons. I refer to the interesting results obtained by the study of the "cathode rays." These rays are emitted from the negative electrode of a Crooke's tube, when worked with an induction coil; they travel in straight lines, can be deflected by a strong

magnet and produce at an object which they meet in their path the same effect which small particles of matter would if moving with great velocity. *Crooke's* hypothesis that these rays were charged gas molecules repelled from the electrode, had to be given up as soon as quantitative measurements of the ratio between the electrical quantity transferred and the corresponding amount of matter were made. Closely connected with these measurements are the names of *Wiechert, J. J. Thomson, Kaufmann, Lenard* and others. Their investigations have led to the result that *in cathode rays this ratio* e/m *equals about* 1.85×10^7, *i. e., the same number as the one found for electrons emitting light.*

Several different methods may be employed to calculate the ratio, the simplest one being the following:

If we have a strong magnetic field of strength H at right angles to the direction of the moving electrons, the force with which the charge is deflected from its rectilinear path equals $H\ e\ v$, where e is the charge on one electron and v its velocity. Acting against this force we have the centrifugal force $= mv^2/\ r$ and a steady deflection is obtained when the two are equal to each other, *i. e.*, $H\ e\ v = mv^2/r$, $r = mv/eH$ where m is the mass of the particle. The electron describes therefore the arc of a circle whose radius is $r = mv/eH$. Now H and r may be measured and thus mv/e found. To eliminate v let an electrostatic and a magnetic field at right angles to each other act at the same time upon the moving electron and adjust the strength of these fields so as to have one just balance the effect of the other. The force on an electric charge e in an electrostatic field of strength F is Fe, therefore under the given conditions $Fe = evH$ or $v = F/H$. Substituting this value of v in the above equation we obtain $v = m/e\ F/H^2$, or $e/m = F/v\ H^2$. Now we have on the right hand side only quantities which we can measure. By this or similar methods a great many observers have found the velocity of the cathode rays to be from 0.3 to 0.9×10^{10} cm/sec or about 1/10 to ⅓ that of light, while e/m by the most accurate methods was found to be about 1.85×10^7 electromagnetic units. The most important result however is that *for different gases this ratio does not change, i. e.*, "the electrons seem to form an invariable constituent of the atoms or molecules of all gases and presumably of all liquids and solids" (J. J. Thomson). To carry a given charge of electricity through an electrolyte requires therefore a mass at least 1000 times greater than if the carriers are cathode rays.

The question whether there are positive electrons has also been investigated. If in a Crooke's tube the negative electrode is perforated, the positively charged atoms pass through these holes and form the so-called "canal-rays." *W. Wien* has found that for these positively charged particles the ratio of the charge to the mass is the same or even smaller than for electrolytes. In other words the canal-rays are produced by "ions" and not by "electrons," the latter being characterized by the large ratio e/m. Real electrons with a positive charge have not been found as yet.

We cannot leave this part of the subject without mentioning other occurrences of electrons. A part of the "Becquerel rays" are of the same nature as cathode rays. These rays emanate as Becquerel first discovered from

certain Uranium salts and they led to the discovery of the new elements Radium (and Polonium?) by Mr. and Mme. *Curie.* These elements send out among others very strong invisible rays which are not stopped even by thin sheets of metal and at first were thought to be similar to "Röntgen rays."

But as soon as *Dorn* and *Becquerel* had shown them to be deflected by a strong magnetic field it became apparent that they belong to the same class as cathode rays. Measurements similar to those described above gave for the velocity of these very penetrating Becquerel rays from 2 to 2.8×10^{10} cm/sec, *i. e.,* almost as large as the velocity of light, while e/m did not differ much from the ratio found for cathode rays. But in this case, and this is of the greatest importance for the development of the electron theory, *Kaufmann has found that the ratio e/m depends for these electrons travelling with such enormous velocity upon their speed,* being the larger the greater the velocity. Another part of the Becquerel rays, namely, those which are easily absorbed and called the -rays, have lately been investigated by *Rutherford* and *Becquerel,* who found their velocity $= 0.2 \times 10^{10}$ cm/sec and $e/m = 6 \times 10^3$. They are therefore very similar to the canal rays and probably positively charged atoms.

Electrons appear however to be much more frequent than in the isolated cases which we have thus far considered. In fact conduction of electricity in gases seems always to be due to the presence of electrons. They may be produced in a number of ways. If you illuminate a charged conductor by Röentgen rays, Becquerel rays, or ultraviolet light, it will lose its charge. Bodies heated to bright red heat or still higher send out electrons. But the most efficient means is through a violent electrical impulse as in the case of cathode rays.

The result of all these experiments is that electrons, or negatively charged particles exist, in which the ratio of the charge to the mass is more than 1000 times larger than it is for a hydrogen ion. A question of greatest importance is: Is the mass of one electron that of an atom, carrying a very much larger amount of electricity than it can in electrolytic action, or is the charge, connected with each particle, the same as that connected with a monovalent ion and is its mass less than the one-thousandth part of that of an hydrogen atom? *J. J. Thomson* has solved this question in a most ingenious way. If a moist ionized, or better, electronized gas is expanded adiabatically the water condenses in drops around the ions as nuclei. If the proper proportion between the water and the ions is chosen the number of water drops equals the number of ions. As *Stokes* has shown, the radius of each drop of water can be measured by the velocity with which a fog formed by the drops sinks down. From the radius and the total amount of water present the number of drops can be calculated. J. J. Thomson found by this method for the charge of each drop, *i. e.,* each electron 1.14×10^{-20} electromagnetic or 3.4×10^{-10} electrostatic units or nearly the same value for each charge as we found for each ion. This charge may therefore be called the smallest quantity of electricity existing, or an atom of electricity; and it may be of interest to know that *M. Planck* has found practically the same number, 4.69×10^{-10} electrostatic units from his theoretical studies on the distribution of energy between

ether and matter. On the other hand it follows that the mass of an electron is smaller than the one-thousandth part of a hydrogen atom. According to J. J. Thomson the so-called ionisation of a gas consists then in a separation of the negatively charged corpuscle, as he calls it, from the remaining part of the atom or molecule, which is left charged positively; in other words positive electrification consists in the absence of these corpuscles from ordinary atoms.

But here the theory cannot stop. A separate mass so much smaller than what we have always supposed to be the smallest and indivisible part of matter, is something so new, that it is natural physicists should have tried to explain away entirely this last trace of matter left in the electrons. And surely in considering the problem before us we must all acknowledge that at least a part of this mass can be only apparent. Let use follow *O. Lodge* in his clear statement of the case, given only a few months ago before the institution of electrical engineers in England.

A moving charged particle is equivalent to an electric current whose strength depends of course upon the velocity of the particle. A magnetic field is set up around an electric current. If the motion starts in *i. e.,* while the charge receives an acceleration, the strength of the magnetic field increases and by electromagnetic induction a counter E. M. F. is produced, which according to Lenz's law acts so as to oppose the action producing the magnetic field, *i. e.,* we have here something which seems to hold the particle back. If the motion is suddenly stopped, induction would tend to continue the motion. We have therefore here something very similar to the inertia of a moving material body opposing mechanical acceleration. But in our case the phenomenon is due entirely to the electromagnetic induction produced by the electrical charge alone, not by mass. It is clear therefore that whatever inertia a moving material body would have due to its amount of matter, it will surely have a larger inertia and therefore a larger apparent mass in virtue of being charged with electricity. *Heaviside* has calculated the energy of a moving charge to be, $Energy = \mu\ e^2u^2/3r$, where μ is the magnetic permeability of the medium, e the electrostatic charge, u its velocity and r the radius of the sphere on which the charge is supposed to be distributed. This kinetic energy measured in mechanical units would be represented by $\frac{1}{2}\ m\ u^2$ where m is the apparent mass, or $\frac{1}{2}\ m\ u^2 = \mu\ e^2u^2/3r$ and $m = \frac{2}{3}\ \mu\ e^2/r$. We may change the expression slightly. The potential of a sphere of radius r in a medium whose dielectric constant is K, is $V = e/rK$, so the apparent mass is $m = \frac{2}{3}\ \mu\ K.e\ e/rK = \frac{2}{3}\ \mu\ K.\ e \times V$. Now the ratio of the electromagnetic unit of quantity of electricity equals 3×10^{10} com. per sec. (*i. e.,* velocity of light) and also equals $\sqrt{\mu K}$. So by denoting the velocity of light by v, we have $m = 2/3v^2\ eV$.

The potential energy of the charge e measured in electrostatic units is $\frac{1}{2}\ e\ V$. But if we have instead of the charge a material particle of mass m moving with the speed of light, its kinetic energy would be $\frac{1}{2}\ mv^2 = \frac{1}{3}\ eV = \frac{2}{3}\ (\frac{1}{2}\ e\ V)$. If a material body having a mass equal to the apparent mass of the moving charge were travelling with the velocity of light, its kinetic energy would be two-thirds the potential energy of the electrical charge when standing still.

Let a sphere be charged to one volt by a quantity of one coulomb. The electrostatic unit of E.M.F. is 3×10^{10} larger than the corresponding electromagnetic unit, but the ratio is just reversed for the units of quantity of electricity, therefore the electrostatic energy corresponding to eV is the same as the electromagnetic energy of one volt-coulomb, namely, 10^7 ergs. Its apparent mass would be $\frac{2}{3}\, 10^7/9 \times 10^{20} = 2/27 \times 10^{-13}$ gr. $= 10^{-11}$ mg., a very small mass indeed. As we saw above, the charge on one ion is 10^{-10} electrostatic units. The radius of an atom is about 10^{-8} cm, so the electrostatic potential of an ion is 0.01 electrostatic units $= 0.01 \times 3 \times 10^{10} = 3 \times 10^8$ electromagnetic units $= 3$ volts. Its apparent mass due to the charge which we may assume as $\frac{1}{3} \times 10^{-19}$ coulombs, is then $m = 10^{-14}.\ 10^{-19} = 10^{-33}$ grams. Comparison with the mass of a hydrogen atom, 10^{-25} grams, shows it is negligible.

In the case of an electron there is however no necessity to assume the radius to be as large as that of an atom, on the contrary it is probable that it is very much smaller than that. Therefore supposing it to be 10^{-13} cm instead of 10^{-8} cm, its potential would be 1000 electrostatic units $=$ 300000 volts, and its apparent mass $m' = 10^{-28}$ grams or about one one-thousandth of that of a hydrogen atom.

The apparent mass of an electron, found experimentally would follow directly from the assumption that the charge is distributed over a sphere of a very small radius and under these conditions, the supposition that the charge is connected to a material particle, is entirely unnecessary. This way of looking at the subject has been strengthened by mathematical deductions and experiments of very recent date. Some time ago it was shown by *Heaviside* that for a charge moving with a velocity smaller than that of light, *e. g.*, the electrons of the cathode rays, its apparent mass will always bear the same ratio to the charge, independent of the velocity, but that the mass would appear larger, if the velocity increases and approximates that of light. Now, as I told you before, the Becquerel rays travel much faster than ordinary cathode rays and reach ¾ of the speed of light. So while the cathode rays show always the same ratio e/m we should expect an increase of the apparent mass with the velocity for the Becquerel rays, and this has actually been found by *Kaufmann.*

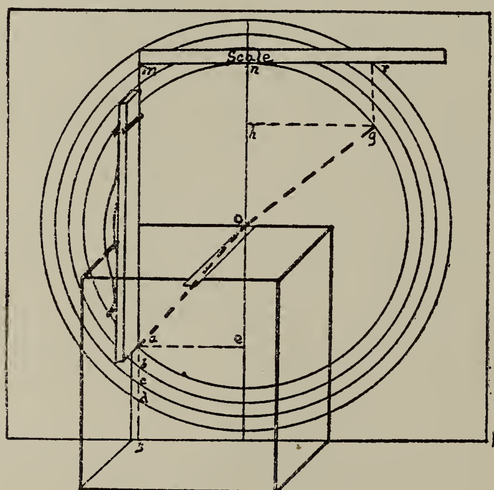
Besides theoretical calculations published a month or two ago by *Abraham,* which were based on the supposition that the so-called mass of the electrons is due only to the inertia effect produced by a rapidly moving electric charge without any mass, led to equations which when compared with Kaufmann's results showed a complete agreement between theory and experiment.

The temptation to extend now these ideas still further is very great indeed. So we must expect in the near future attempts to explain matter as consisting simply of conglomerates of moving electrons, and this would lead us finally to assume as the framework of our universe a continuous ether and imbedded in it the electrons which by the particular manner of combination and motion give rise to the effects we now attribute to matter. The first steps in this direction have been made. Let us hope it is in the right direction and that the electron theory of matter will not in the end prove to be a scientific mirage, which with all our efforts we will never be able to reach.

APPARATUS FOR THE DETERMINATION OF THE INDEX OF REFRACTION OF WATER.

A. O. WILKINSON, DETROIT.

Referring to the accompanying diagram the construction and the working of the apparatus will be easily understood. The apparatus consists of a pine board about 12x18 inches clamped in a vertical position, having several concentric circles drawn upon it. A vertical line is drawn through the center of the circles; to the left of the center, about half of the radius, a line is drawn parallel to the vertical. In front of this board is a rectangular jar whose top is even with the center of the circles. A wooden rod is arranged



to slide inside the jar so that its right hand edge coincides with the line *m s*. A thin metal strip is placed on the jar so that its edge coincides with the center of the circle. The jar is filled with water as full as can be without its couching the metal strip. The lower end of the sliding rod is made to coincide with the intersection of a circle and the line *m s*. By sighting a pin is placed in the same circle so that the end of the rod, the edge of the metal strip, and the pin appear to be in a straight line. The line of the angle of incidence *a e* and of the angle of refraction *h g* can be read on the scale *m r* and the index obtained by dividing the latter by the former. Other trials can be made by using the other circles.

THE INDEX OF THE REFRACTION OF GLASS.

H. N. CHUTE, ANN ARBOR.

For the past sixteen years I have been using what might be termed, by way of distinction, the pin method, of determining the index of refraction of glass. Those who have not tried this method are likely to class it as

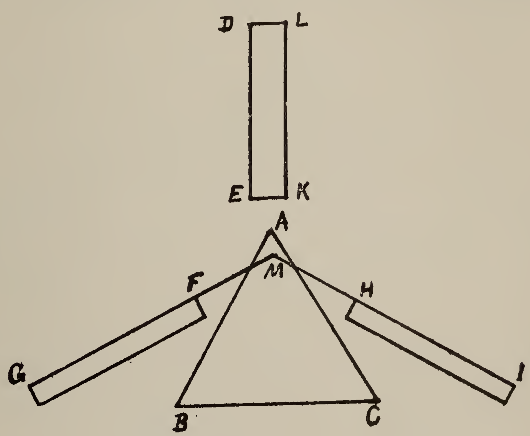


Fig. 1.

crude and not calculated to yield even an approximation. The purpose of this paper is to call attention to the simplicity of the method and to show by an example the degree of accuracy possible. Of late I have been substituting rulers for pins with good effect.

The apparatus needed is a glass prism, three short narrow rulers, a protractor, and a small square board. A sheet of white paper is fastened to the board with thumb tacks.

The first step in the problem is to measure the refracting angle of the prism. The prism, Fig. 1, is placed on end near the center of the board and a ruler, D K, placed at the angle, A, to be measured as shown in the figure. Rotate the board till the light from the window brightly illuminates the edge, D E. Then, place a second straight edge so that its edge, G F, is exactly in line with the reflection of D E seen in the face, A B, of the prism. With a finely pointed pencil draw a line along the edge G F. Now rotate the board to a position in which K L is illuminated and place the ruler H I in line with the reflection of L K in the face A C. Draw the line, H I. Remove the prism and continue the lines G F and H I till they intersect at M. With the protractor measure the angle G M I. Half of this angle will be the value of the angle A in accordance with the principle that the displacement of the reflected ray is twice that through which the mirror surface turns,

since D E may be considered as a ray of light incident on a mirror surface A B which by rotating through an angle A forms the other face, A C, of the prism. The angle G M I thus becomes the angle through which the reflected ray is displaced.

To measure the angle of deviation, mount the prism as before, and place one of the rulers with its edge making an acute angle with the face, A B, of the prism. Fig. 2. Turn the board till the light strikes the edge of the ruler, and then place the second ruler, F G, so that by sighting along F G it seems to be in line with the edge, D E, seen through the prism. The edge,

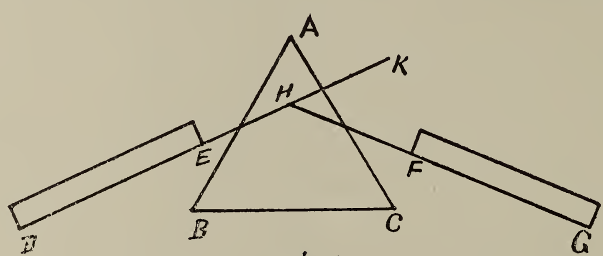


Fig. 2.

D E, of the ruler seen through the prism will be colored with the spectrum tints. Now turn the prism till the edges of the rulers seem to make equal angles with these faces. This may put the rulers a little out of alignment and adjustment of them will be necessary. Draw the lines, D E and F G. Remove the prism and prolong the line G F cutting D E produced at H. With a protractor measure the angle K H G. This will be the angle of deviation.

It is shown in geometrical optics that $\mu = \dfrac{\sin \frac{1}{2}(a + d)}{\sin \frac{1}{2} a}$ in which *a* is the refracting angle and *d* is the angle of deviation. By substituting the value obtained for these angles in this formula, the index of refraction is obtained.

To test the accuracy of the method I applied it to the prism on one of Browning's spectroscopes. The refracting angle of this prism as given by using the instrument to which it belongs is 59° 48′ 20″. Using rulers I obtained the following values:—

119.75
119.50
119.75
119.50
119.50

119.69 ÷ 2 = 59° 48′, showing a difference of only 20″.

The angle of deviation of the F-line as given by the spectroscope is 51° 38′. Using rulers I obtained the following values:—

51.75
51.50
51.75
51.50
51.00
51.25

51.46 = 51° 27′.6, showing a difference of 10′.4, which is in part due to the alignment not being made exactly on the E-line of the spectrum.

Using the spectrometer values the index = 1.657.

Using the method of alignment values the index = 1.656.

The method commends itself for the following reasons:—

1st. The inexpensiveness of the apparatus.

2nd. No dark room is required.

3rd. It is based on the same principles as the spectrometer method, the rulers taking the place of the telescope and collimator.

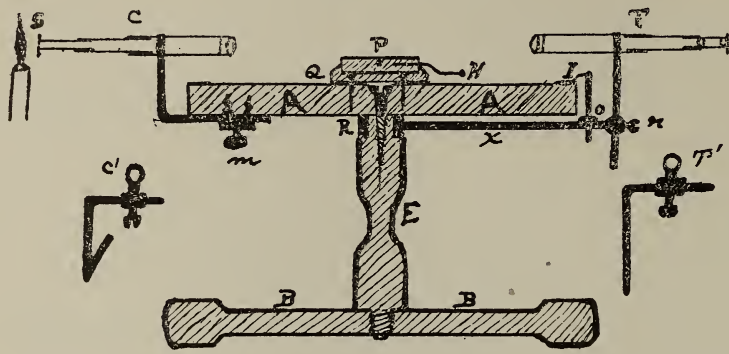
4th. It lends itself to quite a high degree of accuracy.

A SIMPLE SPECTROMETER.

BY CHAS. H. SLATER, PONTIAC, MICH.

A Spectrometer is ordinarily too expensive a piece of apparatus for most high school laboratories to possess. One that answers all the purposes very well may be easily constructed and at a very moderate cost. The base (B B) of the one herein described was the support for an old planetarium long since laid on the shelf. In the absence of one of a similar nature, a base may be easily devised which should be broad and of well seasoned hard wood, the upright (E) being about 12 inches high and 2½ inches in diameter. The top of the upright is turned round 1½ inches in diameter and ½ inch wide. On this shoulder is a closely fitting iron ring (R) into which is screwed a rod (X), the support for the reading telescope (T). The circular disk (A) is thoroughly seasoned walnut 1 inch thick and 13 inches in diameter. A 3-inch screw holds it firmly to the upright (E). A circular scale reading to the fourth of a degree is glued to the rim of disk (A). A prism stand (P) is placed at the center of the disk (A). It consists of a circular plate (P) which rests in a closely fitting piece (Q). An object on the plate (P) can be rotated to any position by the handle (H). The reading telescope (T) has vertical and lateral adjustment so that it

may be easily centered. A small brass pointer (I) is mounted in front of the telescope; it serves the double purpose of a reading index and a partial support for the telescope rod (X). The collimator (c) is mounted on a rod bent twice at right angles (c′) and clamped at (M) in such a position that parallel rays from the right (s) will pass across the center of plate (P).



A collimator is obtained by altering a second reading telescope as follows: The lenses in the eye piece are removed and a small metal slit is placed between the aperture and the small shutter in the eye piece. The focal length of the objective lens being found, the tube is then adjusted to that length, clamped in position at (M), centered, and the spectrometer is ready for use.

As to expense: A blacksmith will do the necessary metal work for $1.00 and good telescopes can be gotten for $2.00 each, but they will serve many other uses, as in timing the pendulum, reading the galvanometer, and for showing the construction of a telescope. Every laboratory should have two at least. The circular scale costs 35 cents. Thus any laboratory that possesses the telescopes may obtain a satisfactory spectrometer at an actual cost of less than $1.50. The usual cost of high school spectrometers is from $20.00 to $40.00.

WHAT TO PUT IN A NOTE-BOOK.

E. L. KEELER, CENTRAL NORMAL, MT. PLEASANT.

This is a very important subject and one upon which all teachers of Physics do not think alike. Some think one thing and some another.

Let me ask—what does the note-book do for the student, or rather what should it do? I believe it trains him to observe carefully and accurately; to think sharply and logically, to express with clearness and precision and to formulate exact statements of his results. It fastens the lessons taught by an

experiment firmly in the student's mind and lastly it furnishes a record for future reference or study.

I believe you will agree with me that these are some of the main objects of a note-book and if so it is a very important piece of apparatus for the Physical laboratory.

It has been my observation and experience that the great trouble with the young people of today is the inability to make a good, clear, accurate statement or record of anything. Think of some of the examination papers you received last week, or some of the letters you are receiving daily and I am quite sure they will bear out my statement.

It is just as essential that the pupil be able to tell in a straightforward, accurate and vigorous way what he has done and how he did it as it is for him to do it at all. The note-book should help the student in getting a clear conception of all that the experiment is intended to convey.

The note-book is as important as the experiment itself, but the manner in which some note-books are kept is worse than nothing from the standpoint of Physics and English. Some are lumbered up with useless material and very important data left out. This is one of the serious troubles, if not the most serious one, with which every teacher has to contend. The student must learn to discriminate between important and unimportant data and detail, and the rightly kept note-book will teach this discrimination.

The first thing to be placed in the note-book is the date of the experiment. This should be followed by the number of the experiment, or a brief statement of it. Following this should be the purpose of the experiment which should be stated in simple, clear language, giving the object of the problem or experiment.

If the apparatus is not described in the manual, I would require the students to do it briefly. This will assist them in understanding the experiment. Often a little more than an illusion to the drawing will be sufficient.

This leads me to the question—what kind of a drawing shall be placed in the note-book? Must it be a work of art? Should it be a perspective drawing or sketch? In short, should it be a picture? Most emphatically *no*. Such a drawing has no place in the note-book. The drawing should be limited to such diagrams as are necessary to explain the work. A mere outline drawing accurately done is all that should be required.

When should this drawing be done and when should the notes be recorded in the note-book? To this question there is but one answer and that is, as soon as they are taken. I am opposed to taking rough notes and then recopying them. This is done in too many high schools. The working of the experiment and making a record of the work should not be separated. These two processes should be made one and I do not consider the experiment is complete until a record of it is made.

There is but one time and one place to make this record and that is in the laboratory and at the time the experiment is performed. I feel that this record must be made before the interest is lost, or the details forgotten. If the record is made in this way, I am sure it is not drudgery. The students enjoy it rather than dislike it because they are never behind with their notes.

Require the students to keep their note-books in the laboratory. They are necessary pieces of apparatus for the laboratory. Make the first notes final. Limit everything to the experiment and allow nothing in the note-book that does not pertain to the experiment. Care must be taken that every measuremet and determination is recorded in such a way that the meaning of the data will be perfectly clear and self explanatory, and that the statements conform to the results found in the laboratory.

The form of the record is a very important question. It often determines the interest taken in it by the pupils. The record should be simple and to the point and this is seldom so unless the teacher gives them a model. A point is reached and often times missed when the student endeavors to state it. I have seen students write a dozen lines to say that the diameter of a wire is five milimeters.

I am in favor of representing the data graphically. This method will show the eye the relation between two quantities which are connected, so that any change in the value of one produces a change in the other. For example:—The relation of volume and weight of different masses of the same substance. In introducing the student to the subject of plotting it is desirable that the relation to be represented should be as simple as possible and therefore I think a straight line should be the first one plotted and this implies that one quantity is proportional to the other while a curve implies that the law connecting the two is more complex. It will be seen on account of errors of observation the points will not all lie in a straight line, or a curve, but that they may be a little to the one side or the other. This will point out where mistakes have been made in observation and show where approximate values would be. Such work as plotting curves can scarcely be overestimated and I believe in general we do too little of it.

The argument and operation should contain the entire mathematical calculation by which the results are obtained and the analysis of the steps taken to secure these results. In this paragraph, I would have the theory of the experiment given and a discussion of the error of the result.

Under operation, 1st—A very brief, concise statement of what was done. 2nd—A careful arrangement of the data. 3rd—Interpretation of the data from which arises the conclusion. When the law is proved the students should see the proof of the law in his figures. Many times I have observed students state conclusions drawn from their own knowledge from the text-book and not from their work in the laboratory, or to state false conclusions. These I have found to be quite common in *qualitative* experiments. It is distressing to a teacher to see a pupil make a neat record and then derive a wrong conclusion, or state something that has no connection to the problem.

Let me say in conclusion, First—that the written work in the note-book, when properly done, is splendid training in English besides fixing the experiment better in the students' mind and that it trains the student to express himself, accurately, logically and briefly. Second—that there is one time and one place for the note-book work and that is in the laboratory where the experiment is being performed or shortly after. Third—that the

drawing should be an out-line drawing and never a picture. Fourth—that much work should be done in representing the data graphically. Fifth, and last—that the argument and operation should contain the entire mathematical calculation which should lead to the conclusion and that the student should see the proof of the law or theory of the experiment in his own work.

HOW TO DISCUSS ERRORS WITH PUPILS.

C. H. SLATER, PONTIAC.

The following question and answer is found in an article in School Science some months ago: "What are the characteristics of the training given by laboratory courses? In the first place, the critical faculty is directly stimulated, the observing powers are in constant exercise, and a wholesome self reliance is cultivated." The first reply to this question brings directly before us the importance of a due consideration of the treatment of errors. The pupil very soon finds that most statements in the book are only approximately true under the conditions of his experiment. He observes the effects of varying the conditions of the experiment. Soon, he discovers that greater care and better apparatus lessen the variations from correct values and as a result he gains confidence in his own work, understands his results better, and determines for himself the degree of dependence to be attached to them.

It will therefore be the purpose of this paper to refer briefly to a few of the points to be considered in discussing "Errors" with one's pupils. In a general way, the subject should be outlined and explained by the instructor at the time their first laboratory directions are given previous to beginning that work. As it proceeds, individual attention can be called to errors as specific cases arise. After a half dozen experiments are completed a part of a period at least could profitably be given up to a further discussion of causes, remedies, and kinds of errors, and the benefit to be derived from a consideration of them. The note-book discussion of each experiment should give a good account of such errors as may occur and their effect on the results. This first discussion of errors should by all means distinguish clearly between errors and mistakes. The former can not be avoided; the latter should seldom occur. The one offers an opportunity for improvement, the other is failure. Mistakes have their origin largely in lack of preparation, together with careless, hurried, and impatient work. As a result, it often becomes necessary at first to give a pupil lessons in arithmetic or reading and to ask for repetitions of the experiment. He also observes a class-mate working near by who is more successful because of previous preparation and careful, thoughtful work. Thus the poorly prepared pupil sees half his time wasted, sees classmates go ahead of him and the lesson of avoiding *mistakes* is soon learned. I find also the requiring of a good permanent record

lessens the number of mistakes. On the other hand errors must occur, but they may be made small and some nearly eliminated. The pupil's task is to point out where they arise, to show their effect on the result obtained, and to explain any device or method by which they may be avoided. To enable them to do this I find the following classification to be helpful, viz.: General, Instrumental, and Personal Errors. The most common may be called General Errors which include such as result from inexperience, limited time, poor light, approximating fractional parts of scale units, etc. These may be eliminated in a large measure by the method of averages and by varying the conditions as much as possible. The use of checks at every available place aids greatly in detecting errors and mistakes, one other thing that might be mentioned as a general remedy because of its very healthy influence on the work is to require the finding of the per cent of error in all quantitative work. These general sources of error and remedies as the name signifies apply to all experiments, while the other kinds are more special and must be treated as they arise. Instrumental and Personal Errors are for the most part remedied by special devices of method and apparatus. The former will include such errors as result from crude apparatus, friction, parallax, Capillarity, radiation, etc.

Personal Errors are such as depend to some degree on one or more of the senses, as on touch in the adjustment of the micrometer screw and Vernier Calipers, or on hearing and sight as in measuring the length of a resonant column of air, or finding the laws of vibrating strings. It certainly would be interesting to discuss at some length methods and devices for eliminating these instrumental and personal errors, but they are so well pointed out in our manuals that it will not be necessary to take them up in this paper. As to the value of a consideration of errors, little need be said to the pupil. To him there are several obvious reasons. He judges the accuracy of his own work and sees the advantage of this or that method under such and such conditions. His ingenuity is exercised, while at the same time constructive and careful habits are formed in overcoming difficulties that arise which hinder him from obtaining the desired results.

(a) WHEN AND WHERE SHOULD THE NOTE-BOOK BE WRITTEN? (b) CORRECTION OF NOTE-BOOKS.

L. M. PARROTT, SAGINAW.

The topic as assigned involves two thoughts. I cannot *answer* either of them. In the few moments allotted me, however, I will but briefly attempt to present the method I have followed and then throw it open for discussion.

Two days before his laboratory period each pupil is told the number of the experiment he is to work, and is expected to have both himself and note-book prepared when he enters the laboratory, so that he can immediately attack his problem by a straight from the shoulder method. At the begin-

ning of the laboratory period he is handed one of the printed forms indicated below, and all his data is recorded in duplicate, in his book and on this form, exactly as taken. This duplicate is left with the teacher, all the calculations being indicated below the recorded data and as much worked out as time permits. Then, during the coming week, each pupil in the quiet of his own home must write a discussion of his experiment, bringing out its purpose, the principles involved, his observations, manipulations, conclusions, etc., in the very best literary style of which he is capable.

SAGINAW, EAST SIDE, HIGH SCHOOL.

PHYSICAL LABORATORY.

........................190.. Experiment No...........

APPARATUS :
.... No...........
.... No...........
.... No...........
.... No...........

Calculations:

....

These data sheets enable the teacher to keep in close touch with each student's progress, and a daily record of what has been attained. Each is carefully looked over, the character of the data taken and the indicated calculations outlining to a large degree what the pupil has acquired from the experiment. Once a month the books themselves are put in the teacher's hands and the discussions are carefully read, marked, and points noted for consultation with the pupil.

A METHOD FOR DETERMINING THE DENSITY OF GASES.

F. R. GORTON, YPSILANTI.

That air has weight may be ascertained by the use of an incandescent lamp bulb and a balance that turns easily with 0.1 gram. The bulb is first counterpoised by means of shot or sand. Air is then admitted by inserting a nail in the neck of the bulb and tapping it lightly with a mallet. The end of the tube C shown in the figure is thus broken off and every fragment caught in the bulb. On replacing the bulb upon the balance a gain of nearly 0.2 gram is shown.

In order to ascertain the density of air, it is advisable to remove the brass fixture attached to the bulb and clear away every particle of cement. The bulb is then weighed as accurately as possible. Dry air may be collected in a small gas bag, from which it is admitted into the bulb by a T-tube arranged as shown in the figure. The rod D extends through the tube projecting at the ends. The joint A is made air tight by means of rubber tubing. At B a piece of tubing causes the glass tube to make an air-tight connection with the bulb N. If the inlet pipe is now connected with the gas bag, a tap upon the rod at D will break the bulb at C and admit air from the gas bag. The gain in weight having been found, the volume of air admitted is determined by filling the bulb with water of known density and weighing.

The density of other gases is found by filling the gas bag with the dry gas whose density is required and admitting it in the same manner.

AN EXPERIMENT FOR ILLUSTRATING OSMOTIC PRESSURE.

C. F. ADAMS, DETROIT CENTRAL HIGH SCHOOL.

It is well known that the walls of many plant cells form "semi-permeable" membranes. Such membranes allow diffusion of certain substances through them in one direction while not permitting diffusion in the opposite direction. Dr. Louis Murbach, of our school, first called my attention to the use of the carrot and other similar vegetables for the purpose of illustrating osmosis.

To perform this experiment a hole is bored along the axis of a large, sound carrot, about 10 cm. deep and 2.5 cm. in diameter, a carpenter's bit being used for the purpose. I also peeled the carrot up to its crown, although I suspect that this is unnecessary. The cavity in the carrot is then nearly filled with sugar and water added to it to within a centimeter of the top. To close the cavity a two-hole rubber stopper is used, in one hole of which is inserted a long piece of barometer tubing having a bore of about two millimeters. After the stopper is driven firmly into the carrot the other hole in

the stopper is closed by a glass plug. There should be sufficient water in the carrot to entirely fill the cavity so that the insertion of the glass plug will force the water a centimeter or so up the tube. The carrot and tube are then supported by a clamp so that the carrot stands entirely submerged in a dish of water. If care has been taken to put the apparatus together so that there is no chance for leakage the liquid will immediately begin to rise in the tube.

At my first attempt with this experiment the liquid rose to the top of a tube 130 cm. long in two hours and a half. I filled the same carrot again lengthening the tube to 308 cm. and allowed it to stand during the night. In the morning the current was still upward while a trail of thick syrup on the outside of the tube showed that the sugar solution had run over during the night. I again connected the same carrot by a U tube to a bottle filled with mercury and the mercury was forced up a tube 63 cm. when on my attempting to move the apparatus a leak occurred and put an end to the experiment.

As the osmotic pressure of a saturated solution of sugar is known to be several atmospheres I would suggest that one might in this way force the mercury up to a considerable height. Possibly it would be found advantageous to bind the carrot with cheese-cloth to prevent its bursting, and the stoppers would certainly need to be fastened in place. I hope to hear that someone has taken the matter up and secured interesting results.

GAS GENERATOR.

PRINCIPAL JOHN P. EVERETT, PONTIAC.

When I began teaching physics at Grass Lake I met with a difficulty usually encountered in small places—no gas for laboratory purposes. The matter seemed of sufficient importance to warrant some attention, and the apparatus herein described is the result. For the main ideas embodied in constructing the generator I am indebted to Dr. Chadwick. My work was rather one of adaptation than of invention.

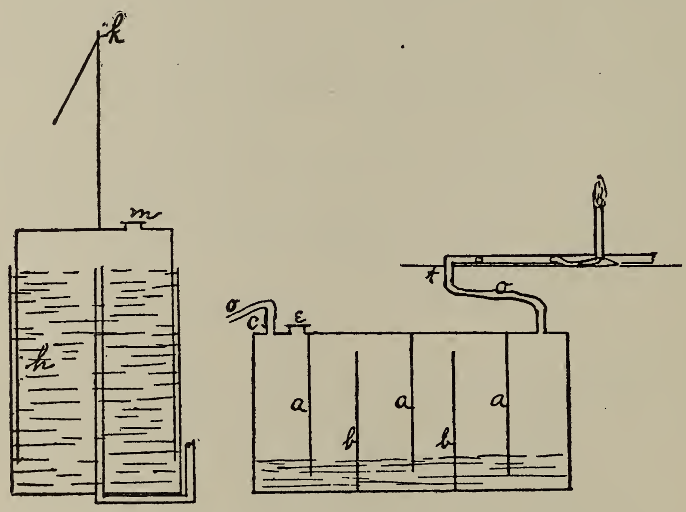
The operation of the generator depends upon the fact that an inflammable gas is generated by pressing air through gasoline. The actual generator consists of a galvanized iron box 12 in. x 10 in. x 7 in., a longitudinal section of which is shown in the accompanying diagram. Let down from the top and soldered to the sides are three partitions (a) which reach within half an inch of the bottom. Two partitions (b) extend from the bottom to within half an inch of the top. Gasoline is introduced by means of an air-tight cap (e). The bottom should be covered from one to two inches. Even distribution is secured by tipping the generator on its side and righting it by turning on its longitudinal axis. If air is forced in at (c) it must pass through gasoline three times and emerges at (d) a highly inflammable gas.

The required air pressure at (c) is afforded by the common arrangement of two cylindrical tanks (h), the lower being filled with water. I used

tanks about three feet in diameter and of a height equal to twice the width of galvanized sheet iron. The upper tank must be weighted. A triple pulley (k) fastened to the ceiling was used in raising the tank. Air is admitted as the tank rises by removing the cap (m).

For conducting the gas from the generator I used a half-inch gas pipe (t) running along the back of a bench. Into this pipe were let ordinary air cocks for attaching the rubber tubing leading to the burners. Since the generator must be moved each time it is fillled, connection with it is made by flexible rubber tubing (o).

The flame is quite similar to that obtained from ordinary coal gas. It can be made luminous or non-luminous at will. The Bunsen burners, however, with some modifications, can be made to work very well. The opening through which gas is admitted to the base of the burner must be enlarged and the holes at the side of the tube for admitting air should be quite small. This I accomplished by slipping a perforated paper between the movable collar and the tube.



This generator is to be commended (1) for its cheapness. Twenty dollars will provide an elaborate apparatus, while a serviceable one can be constructed for much less. (2) It can be made at any tin shop. (3) It is safe. The flame cannot be drawn back to produce an explosion. I demonstrated this by experiments in the open air. (4) Its operation is economical. One gallon of gasoline will last for weeks. (5) There is no large volume of gas stored to leak and cause trouble, and no gas is generated except as needed. The apparatus is rendered absolutely inoperative by tying to a peg the rope leading from the pulley (k).

Note.—Since the meeting of the Schoolmasters' Club I have learned that burners are now manufactured which are especially designed for the gasoline flame. They can be obtained of houses dealing in laboratory supplies.

COEFFICIENT OF LINEAR EXPANSION.

C. S. COOKE, DETROIT CENTRAL HIGH SCHOOL.

The idea for the form of apparatus herein described was suggested to me by N. H. Williams, Shortridge High School, Indianapolis. After experimenting for some time with different pieces of apparatus for measuring the coefficient of linear expansion, I can safely recommend the one described below for its simple construction, small cost and reasonable accuracy, considering the several sources of error incumbent on any coefficient of expansion piece. Where there are several similar pieces of apparatus to be set up for a laboratory section, the time saved by having apparatus of simple construction is of considerable importance. With most pieces some electrical device is used for obtaining the expansion. Such devices are often unreliable in the hands of the pupil. Often times the frame work for supporting the metal tube or rod will expand with heat or moisture, thus introducing errors into the result. In this piece there are no troublesome electrical contacts and it is comparatively free from the other difficulty mentioned. With the piece described it is possible for the pupil to obtain several readings during the laboratory period. We have observed that results obtained by pupils are uniformly better than with any other apparatus we have tried.

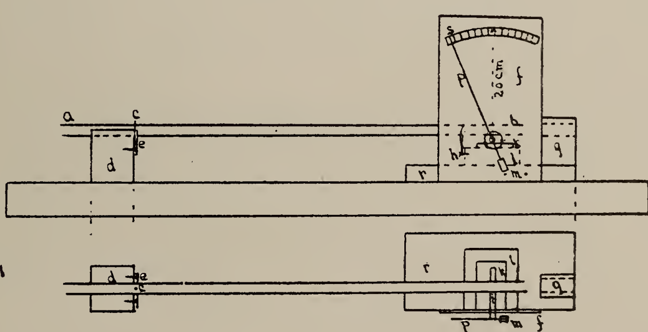


Fig 1

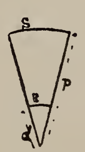


Fig 2

The apparatus consists of a brass tube (ab) about one meter long (Fig. 1) whose expansion is to be measured. One end of the tube rests in a grooved block (d) while the other end rests on a small brass roller (i) which carries the slender pointer (p). (m) is a counterbalancing weight. The roller rests on a piece of ground glass (k) which in turn is mounted on a block of wood (1). (1) is fastened to a small board (r). The ground

glass prevents slipping of the roller. (f) is a piece of cardboard or sheet metal fastened to the baseboard (r). The pointer moves in front of a scale (s) graduated in centimeters. The blocks (d) and (r) are firmly clamped to the table. A piece of gas tubing connects at (a) with a steam or cold water supply. The tubing at the other end passes through a hole in the block (g). This is to permit handling of the outlet tube during the experiment without disturbing any adjustments. The brass tube is kept in firm contact with the roller by a rubber band fastened to a tack at (h). The expansion takes place from the fixed point (c). A small pin passes through the tube and projects about 2 cms. of an inch from the under side and is securely clamped to the block (d) by a brass piece (e).

As the tube expands, the amount of expansion is measured off on the surface of the roller. This expansion bears the same relation to the movement of the end of the pointer that the diameter of the roller bears to the length of the pointer; thus in figure 2, E : S : : d : p. The shifting of the center of rotation introduces no error that is measureable.

We have tried covering the tube with a jacket to prevent errors by radiation, but do not find that the precaution is of any particular value.

The following tabular form shows the method of recording data and calculating results:—

Length of tube in cm.	83.4 cm.
Diameter of roller (measured with micrometer screw)	.2402 cm.
Temperature of the water (taken at outlet)	3°.8 C.
Temperature of steam (computed from barometer)	99°.72 C.
Change of temperature	95°.92 C.
First position of the pointer	1.00 cm.
Second position of the pointer	13.49 cm.
Change of position of the pointer	12.49 cm.
Expansion (x : 12.49 : : .2402 : 20)	.1500 cm.
Expansion per degree	.001574 cm.
Expansion per degree per unit of length	.00001887 cm.
Accepted coefficient of expansion	.00001875 cm.

Error .00000012. Percentage of error, .6%.

SOME PHENOMENA OF HIGH FREQUENCY CURRENTS.

BENJAMIN F. BAILEY, ANN ARBOR.

The fundamental principle upon which the operation of high frequency apparatus is based, is the fact that the discharge of a condenser is usually oscillatory in character. In other words, if a condenser is charged and then allowed to discharge through a piece of wire, the current instead of flowing

steadily in one direction until all the energy is expended, oscillates rapidly back and forth a number of times. The case is precisely analogous to that of a pendulum or of a spring suddenly released from pressure.

Thus in Fig. I and II if we imagine an electric current, flowing in the circuit of I, and the pendulum in motion in II, the analogy between the cases is very striking. Thus the electro-motive force, driving the charge into the condenser (equal to Q C) corresponds to the force pushing the pendulum to one side (equal to $k\ s$). The back force due to acceleration $m\ (d^2q \div dt^2)$ takes the place of the back *E.M.F.* due to self-induction $L(dq^2 \div dt^2)$, and the force due to friction $K_2v = K_2(ds \div dt)$ that of the *E.M.F* due to resistance $RI = R(dq \div dt)$. Indeed the equations for the two phenomena are identical, and we can calculate the period of vibration of one as easily as that of the other. The theory is too long to give here, and I shall merely give two results of it.

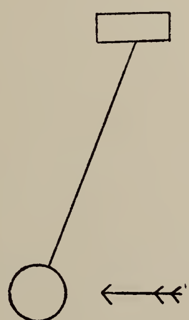


Fig. II.

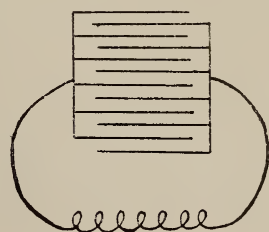


Fig. I.

One is that the period of one complete vibration of such a system is given by $T = 2\pi\sqrt{LC}$ comparable with $T = 2\pi\sqrt{(e \div g)}$ in the case of a pendulum), and the other is that in order to have any oscillation at all, it is necessary that the resistance be less than $2\sqrt{(L \div C)}$ just as in the case of the pendulum no vibration at all will take place if the viscosity of the medium in which it moves is too great.

The arrangement of the apparatus may be made in various ways, but the most usual set-up is as shown in Fig. III. T. is an ordinary alternating current transformer, charging a current of 50 volts and 120 complete cycles per second, to one of about 5000 volts, and of course the same frequency. This charges the condenser *C* with a certain quantity of electricity 240 times per second. In Fig. IV let the sine curve represent the variations of *E.M.F.* across the terminals of this first step up transformer. At the point *a* there is no *E.M.F.* applied to the condenser and consequently no charge in it. As the voltage increases along the curve *a e* the charge in the condenser also increases, and we finally come to the point *b* where the voltage is great enough so that the air gap *g* (Fig. III) breaks down, and allows the condenser to discharge through the primary *L* of *a* second step up transformer T_2.

This circuit of the condenser the spark gap and the primary of the second step up transformer is like that of Fig. I except for the resistance of

the spark gap and the discharge is consequently a high frequency current. This current is represented by *bdc* in which the length of the oscillations is greatly exaggerated on account of the difficulty of drawing them sufficiently close together. The number is probably greater than is shown there. From the secondary of this transformer we can draw a current like that in the primary except that the voltage is increased about 50 times, and the current decreased in the same ratio. This gives us in this particular machine a current of a frequency of 435,000 double vibrations per second, a voltage at the highest ordinate of 350,000 volts, and an average current of about half an ampere. This is a far greater output than can be obtained from an ordinary induction coil of the same size. The spark discharge produces a regular roar, so great is the volume of current.

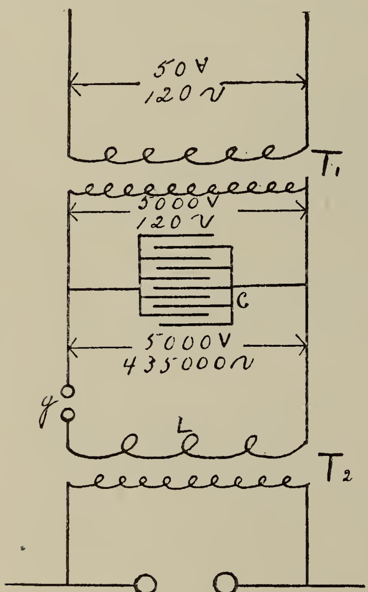


FIG. III.

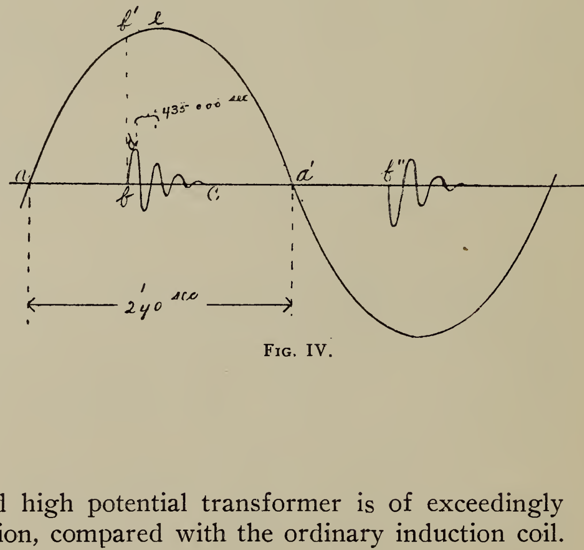


FIG. IV.

The high frequency and high potential transformer is of exceedingly curious and simple construction, compared with the ordinary induction coil. The primary consists of only four turns of heavy copper strip. This is wound upon a frame of wood, somewhat like a bird cage, only with fewer bars. The secondary has only a single layer of 200 turns wound upon a frame within the other one. Owing to the high frequency no iron core is needed; indeed its presence would be a serious detriment. Only about 350 feet of wire is used for the secondary, in comparison with about five miles which would be used in an ordinary coil of like voltage. The whole is immersed in oil to provide good insulation.

There is some liability of an ordinary alternating current arc forming across the gap *g*, and to prevent this a motor is used to rotate a toothed disc to break the arc., An air blast is sometimes used for the same purpose. The coil works nearly as well, however, with a plain spark gap.

We can now examine a little the results of our two equations. Taking the one for the minimum value of the resistance, $R < 2\sqrt{(L \div C)}$ and substituting the value of the self-induction and the capacity, we reach the interesting conclusion that the R is less than 17.2 ohms. Thus the actual resistance of the spark gap is certainly less than 17 ohms and is probably much smaller yet. To figure the frequency is quite simple. We have merely to substitute $T = 2\pi\sqrt{(L \div C)}$ and get for a result $T = \frac{1}{435{,}000}$ of a second or we have 435,000 complete vibrations per second. This enormous frequency is the distinguishing characteristic of the current, and is the cause of many strange effects.

Perhaps the most curious of these are its physiological effects. If we pass an ordinary alternating current through the human body, all the average person can endure without excessive pain is about 0.015 of an ampere. The resistance of the body from hand to hand between dry metal electrodes is about 2500 ohms. Then since the power expended is given by T^2K we have $0.015^2 \times 2500 = 0.56$ watt. In other words about half a watt of power applied continuously in the form of an alternating current of ordinary frequency is about all the human body can endure. What do we find to be the case with the high frequency current? As far as I can find nobody has ever constructed a machine capable of giving all the current a person can endure, so what the limit is we do not know.

The largest amount anyone has used as far as I can find out is 1.5 amperes, which is the maximum amount Elihu Thompson could obtain from his large machine. This means $1.5^2 \times 2500 = 5600$ watts or just one thousand times the amount of power that can be endured in the form of a low frequency current.

This is certainly a very surprising result, but it is not the worst of the case. By reference to Fig. IV it will be seen that there is an interval from c to b'' during which there is no current. Now the 1.5 amperes is the average (strictly speaking the $\sqrt{\text{mean}^2}$ value, but practically the average) value of the current during the whole time, and it is obvious that during the time from b to c it must be larger than this average. Still more, if we take the maximum ordinate as at d, it will be very much greater than 1.5 amperes. Just how much greater this is it is hard to say, but in any particular case we can approximate it so as to get at least an idea of the ratio. The argument is too long to give here but it leads to the surprising result that with an average value of 1.5 amperes the maximum value of the current is probably about 21 amperes. There are numberless theories regarding the harmlessness of these currents, but nothing is certainly known about it. The fact simply remains that there is not time between the very rapid oscillations for the muscles to respond and produce the painful contraction which is characteristic of the alternating current of low frequency. Also there is no electrolysis which is probably responsible for the painful effects of the direct current.

Another surprising property of these currents is the facility they give for what might be called "wireless lighting." If a glass globe, exhausted to the proper degree (a common incandescent lamp bulb will do, but a Geisler tube is better), be held near the high frequency coil, it will light up in much the same manner as though the discharge from an induction coil or a static machine had been passed through it. There need be no connection whatever between the tube and the coil. This is easily understood if we consider what would happen if we held a loop of wire in the same position. Some of the lines of force from the transformer would pass through the loop, and by their rapid change of direction would induce oscillating currents in the loop. Precisely the same thing occurs when we hold a bulb of rarefied gas there. Oscillating currents are induced in the gas in the same way, and it is these currents that cause the gas to light up.

This is an illustration of electro-magnetic induction. The same result can also be obtained by electro-static induction. If we connect the two discharge rods to two metal plates, suspended about a foot apart, we create between them a rapidly alternating electrostatic field of force. This is here called electro-static by courtesy only as it is in reality far from static. If we now hold or suspend a globe of rarefied gas between them, at one instant one plate will be positive, the other negative. By the law of attraction the + plate will attract — electricity to the end of the tube nearest it and repel + electricity to the other end. An instant later (in the case of the machine referred to $\frac{1}{870,000}$ of a second later) the conditions will be reversed and all the electricity attracted to the left of the tube must rush over to the right, and that on the right go to the left. This continues and thus an oscillating current is set up in the tube which causes it to light up.

As regards the application of high frequency currents in practical engineering, the outlook is not encouraging. There is no doubt but that such a system could be made to work (using the ordinary incandescent lamps in the usual manner, and not as described above), but several disadvantages would appear. The high frequency would make trouble on account of the capacity and induction effects, and the regulation would consequently be bad. The only saving would be in the cost of transformers, which, as was said before, would need no iron and very few turns of wire, and consequently could be built very cheaply. This is a comparatively small matter, however, and there is no likelihood of such an application in the near future.

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